Fast neighbor positioning and medium access in wireless networks with directional antennas

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Abstract
We study the problem of fast neighbor positioning and medium access in wireless networks with directional antennas. In this problem, the cross-layer dimension inherently comes into stage through the impact of PHY-layer antenna directionality on medium access. Fast neighbor positioning reduces the network initialization overhead and leaves more time for executing other protocols. Fast medium access leads to larger volume of transmitted data per unit of time. The two problems are studied in a unified manner in a system with one Access Point (AP) and multiple users around it. The AP sequentially scans the space by forming directional beams and applies contention-free or contention-based user polling within each beam. In the former method, polling messages are addressed to a specific user. In the latter, users in a beam contend to have their message received by the AP.

We explore the impact of the contention resolution protocol and the directional beam width on user positioning and medium access delay. A large beam width incurs large expected delay for contention resolution due to the larger expected amount of contention in the beam, but on the other hand, it implies that fewer beams, and hence smaller delay is needed to scan the entire space. We obtain analytic expressions for the total average user positioning and the medium access delay, and we present an optimization method for minimizing it by appropriately selecting the beam width and the persistence probability of the collision resolution protocol. Our method uses accumulated knowledge from previous scans to estimate the anticipated amount of contention in upcoming scans and to adjust the beam width and persistence probability accordingly. Our numerical results demonstrate the efficiency of our techniques in terms of fast neighbor positioning.

1. Introduction

Directional transmission through electronically steerable beams, sectorized antennas or adaptive antenna arrays leads to increased spatial reuse of radio resources and thus high throughput capacity, by allowing the simultaneous transmission and reception to/from multiple users in the same conventional radio channel. Space division multiple access (SDMA) with an adaptive antenna array at the AP enables intra-cell channel reuse by several spatially separable users, by pointing a beam towards the direction of a user and by nulling out interfering users. It also extends the transmission range and reduces interference by focusing the transmission energy in certain directions, instead of performing omnidirectional transmission.

A major challenge in wireless networks is to ensure that each node knows its neighbors. Knowing one's surroundings is an essential first step for various network operations and protocols such as physical layer (PHY) transmission control, medium access, routing or transport
control. When omnidirectional transmission is used, it is sufficient for a node to know the link gain to each of its neighbors; this can be achieved by e.g. pilot symbols. An important class of neighbor discovery protocols with omnidirectional transmission is that of birthday protocols [1]. Each node is in state Transmit (T), Listen (L) or Stand-By (S) in each slot, and its state evolves independently from slot to slot. Two nodes X and Y within range of each other discover each other if Y is in L state, and X is the only node in T state within range of Y.

To reap the benefits of directional transmission, a node needs to know the angular position of each neighbor, which is captured by its spatial signature vector, so as to point its transmission towards each neighbor. In reception mode, the AP can obtain the spatial signature of a neighbor by making use of the preamble of received packets. The preamble is used by the AP to train the antenna array to the spatial signature vector and to compute beamforming weights that effectively steer the main lobe of the beam towards the intended user. Neighbor discovery and positioning methods often require a medium access control protocol to operate. In [2], the authors describe a slotted Aloha system with an adaptive array at the Base Station (BS) which forms one beam. By exploiting a pseudo-random sequence in the packet preamble, the BS computes a beam and locks onto the first received packet in a slot, while nulling out subsequently received packets. A similar method with multi-beam capabilities is presented in [3]. The BS again uses packet preambles to form a beam for each received packet from a different user, and several users are captured. Uplink access to a base station with an antenna array through a modified carrier sense multiple access (CSMA) protocol is addressed in [4].

In transmission mode, the BS can request information about the spatial signature of a user by broadcasting a polling message intended for that user in the downlink [5]. Upon reception of the poll, the user transmits a given sequence of symbols. The BS measures the received signal and uses it to compute the spatial signature and gradually steer a beam towards the direction of the user.

There has been a lot of work on IEEE 802.11-compatible medium access control with directional antennas [6–9]. The key idea is the use of a directional Network Allocation Vector (D-NAV) for notifying neighbors about deferment of transmissions that would hinder ongoing ones in the same locality. If a node receives a request-to-send (RTS) or clear-to-send (CTS) message from a certain direction, it needs to defer only from transmitting towards that direction. The authors in [6] also use successive directional RTS messages to inform neighbors about intended transmissions. The work [10] presents a suite of protocols for IEEE 802.11 networks of nodes with directional antennas that include directional access control, neighbor discovery, link characterization and packet forwarding. Neighbor discovery in systems with directional antennas is an integral part of communication that precedes access control and other network operations [11–14]. The work in [11] presents two classes of probabilistic neighbor discovery algorithms with the goal of maximizing the probability of discovery within a given time interval. Neighbor discovery algorithms based on direct neighbor response or gossip information from other nodes are proposed. A comparison of random and scan-based neighbor discovery methods is presented in [14].

1.1. Our contribution

Fast neighbor positioning allows more time for execution of other protocols and for reducing network overhead. Fast medium access leads to larger volume of transmitted data per unit time. These problems inherently require a cross-layer joint treatment due to the impact of PHY layer antenna directionality on medium access control. The two problems are studied here in a unified manner through directional polling methods with which the Base Station or Access Point (AP) sequentially scans the space by forming directional beams and applies contention-free or contention-based polling within each beam. In the former, polling messages are addressed to each specific user. In the latter, users illuminated by a beam contend to have their message correctly received by the AP. Polling messages aim at locating users in the user positioning phase, and at initiating contention among users that have data to send at the medium access phase.

In order to demonstrate the benefits of our approach, we consider a simple contention resolution algorithm within each beam. We explore the impact of contention resolution parameters and the directional beam width on user positioning and medium access delay. A large beam width incurs large expected contention resolution delay in the beam due to the larger expected number of users. On the other hand, a large beam width implies that fewer beams (and hence smaller delay) are needed to scan the entire space. We obtain analytic expressions for the total average user positioning and the medium access delay, and we present an optimization method for minimizing the delay by appropriately selecting the beam width and the persistence probability of the collision resolution protocol. Our method uses accumulated knowledge from previous scans to estimate the anticipated amount of contention in upcoming scans, so as to adjust the beam width and persistence probability accordingly.

The contribution of our work is as follows: (i) we study the class of directional contention-based and contention-free polling methods with successive scanning of space for user positioning and medium access, (ii) we obtain analytic expressions for user positioning and medium access delays (iii) we formulate and solve the optimization problem of minimizing delay by appropriately selecting the beam width and the persistence probability of the contention resolution protocol that affect contention resolution delay, and (iv) we compare the performance of contention-free and contention-based methods with that of omni-directional and directional transmission. The contention-based and contention-free methods were introduced in the preliminary work [15], and their performance in the presence of user mobility was numerically evaluated in [16]. In this paper, we derive analytic expressions for the delays as in (ii) above, and we present a systematic methodology for optimizing delay as in (iii) above. The rest of the paper is organized as follows. In Section 2 we provide the model and assumptions, and in Section 3 we formulate and solve the optimization problem of minimizing user positioning.
and medium access delay. Numerical results are presented in Section 4. Finally, Section 5 concludes our study.

2. System model

2.1. Directional transmission

We consider one Access Point (AP) and N users around it. We assume that the AP knows the number of users and their identities, but it does not know their locations. If the number of neighbors N is not known, N may stand for a known upper bound on the number of neighbors. We consider uplink medium access by users for transmitting data to the AP. The AP is equipped with a uniform linear array of M antennas, while each user has an omnidirectional antenna. The location of user i = 1, ..., N is captured by its spatial signature vector \(a_i\). The AP and all users are assumed to be in the same elevation angle, so that users have zero azimuth angle with respect to the AP. In line-of-sight (LOS) transmission, vector \(a_i\) is mapped exactly to the physical angular location \(\phi_i\) of the user. In the general case of multi-path, it is a superposition of components, each corresponding to a direction of a path,

\[
a_i = \sum_{i=0}^{L-1} \beta_{ij} \mathbf{v}(\phi_{ij}),
\]

where \(L\) is the number of paths in the multi-path, \(\phi_{ij}\) is the angle with which the \(i\)th path of user \(i\) impinges on the array (with \(\phi_{i0} = \phi_i\) for the LOS path), and \(\beta_{ij}\) is the combined path loss and fading coefficient of the \(i\)th path of user \(i\). Also, \(\mathbf{v}(\phi)\) is the antenna steering vector or antenna response vector at direction \(\phi\), whose \(m\)th component is \(\exp[j(M-1)/2 \cos \phi]\), where \(\lambda\) is the communication wavelength and \(d\) is the distance between two consecutive antenna elements.

The AP has one transceiver and can form one directional beam at a time. A beam is specified by an \(M \times 1\) complex vector \(\mathbf{w} = (w_1, \ldots, w_M)\). The radiation pattern of a beam consists of a main lobe which is centered at a given location and carries the main portion of power, and some side lobes. In order to simplify the subsequent analysis, we adopt an idealized model, where a beam is approximated by a sector of angle \(\theta\) rads, at angular position \(\delta\) with respect to a reference direction (Fig. 1). Thus, the space is covered by \(2\pi/\theta\) beams. The transceiver can control the beam width at will.

The broadcast range of the AP is specified by the maximum transmit power when the array operates in omnidirectional mode. In general, users can be in or out of broadcast range of the AP. Due to mobility, they can move in or out of range at different time instants. In this work, we consider a static snapshot of the system, where users remain either in or out of the broadcast range for the duration of interest.

For ease of exposition, we present the problem here under a coordinated setting with an AP and multiple users around it. Nevertheless, an identical problem arises in ad hoc settings, where a node needs to position its neighbors.

2.2. Spatial signature acquisition for user positioning

To reap the benefits of directional transmission, the AP needs to position users, i.e. to obtain information about their spatial signatures, so as to employ beamforming when transmitting to them. The AP can obtain information about the spatial signature of a user by one of the following methods:

- **Contention-based polling.** The AP sends a beacon polling message inquiring about the existence of any user in the AP transmission range. This message is not intended for a specific user. If the message is received by only one user, the user responds by sending a known sequence of bits, and the AP resolves its spatial signature. The AP uses these bits to train the array, so as to steer the beam towards the direction indicated by the spatial signature of the user. If the message is received by more than one users, their simultaneous responses collide at the AP. The AP then initiates a contention resolution procedure to resolve users and obtain their spatial signatures.

- **Contention-free polling.** The AP first sends a polling message that contains the ID of the intended user. Upon receiving the poll, the user responds by sending (with omnidirectional transmission) a known sequence of bits...
in the uplink. The AP uses these bits to train the array so as to steer the beam towards the direction indicated by the spatial signature of the user. The AP repeats this polling-response method to acquire the spatial signature of each user. This method is referred to as contention-free polling since it does not involve user contention.

The AP can send the contention-free or contention-based polling messages with omni-directional (broadcast) or directional transmission with different beam widths. With broadcast transmission, the AP can resolve users that reside within its broadcast range, while with directional transmission, users out of broadcast range can be resolved as well. Also, since multiple paths exist in the multi-path profile of each user, at least one path should be covered by the beam so that the user receives the polling message.

2.2.1. Contention-based directional polling

In contention-based directional polling, the AP scans the horizon by forming successive directional beams. The AP resolves all users within a beam before proceeding to the next beam. Time is divided into time slots. Each time slot consists of a control message interval and L mini-slots that serve as collision resolution slots. We assume that the local AP and user clocks are synchronized.

At the beginning of a new time slot, the AP sends a polling message by forming a beam of some beam width at a certain direction. The polling message does not contain the address of any user. Each user that is illuminated by the beam receives the message and responds at the beginning of mini-slots by sending back a polling acknowledgment (P_ACK) message that contains a preamble and the user address. If only one user happens to transmit such a P_ACK in a mini-slot, this is received correctly by the AP, and the spatial signature of the user is obtained with the aid of the preamble. In that case, the AP informs the user at the end of the respective mini-slot that its spatial signature is known by sending to it an ACK message with its address, so that it will defer from transmitting P_ACK in subsequent mini-slots. However, if there are multiple users in the beam, their simultaneous P_ACK responses will collide at the AP. The AP will not issue an ACK, and thus users are informed about the collision and upcoming contention. The duration of the mini-slot is at least as large as it suffices to accommodate a P_ACK and an ACK message.

We adopt a very simple method for resolving collisions. Each unresolved user (namely a user that has not received an ACK at the end of previous mini-slots due to collision) retransmits with a persistence probability \( p \) at the beginning of subsequent mini-slots, until eventually it is resolved. This is repeated for the remaining unresolved users, until the AP has indication that all users in the beam are resolved. This happens if no user sends P_ACK at the beginning of a mini-slot. When all users in the beam are resolved, the AP proceeds to formation of the next beam in the next time slot. If the L mini-slots of a time slot expire and the AP does not have any indication that all users have been resolved, the collision resolution procedure continues in the next time slot and the AP sends a polling message again. The stages of collision resolution are depicted in Fig. 2. It can be seen that the beam adjustment is performed on a time slot scale, since it relies on more complicated circuitry operations for directionality adaptation, while the contention resolution is done on a mini-slot scale.

The contention resolution protocol was intentionally kept simple, so that emphasis is placed on its impact and the impact of the beam width on user positioning delay. This contention resolution protocol is essentially parametrized only by the persistence probability, and hence it leads to a tractable optimization problem, as will be discussed later. Note however, that this protocol is reminiscent of the alternation between the Contention Free Period (CFP) and the Contention Period (CP) in the Point Coordination Function (PCF) mode of the IEEE 802.11 protocol [17]. In the PCF mode, the AP first sends a beacon frame announcing that a CFP period is about to begin. Next, the CFP begins and the AP sends the polling message, and then the CP begins, and terminals contend for accessing the medium.

It should be noted that the overhead of the contention resolution algorithm may become significant, particularly due to the amount of empty mini-slots where no user will respond to the poll, depending on the chosen persistence probability. A solution to mitigate such an overhead could be to have the AP send back a message at the end of the mini-slot, denoting that no P_ACK was received in that mini-slot. Users could then respond to this AP feedback by increasing their persistence probability in the next mini-slot.

2.2.2. Contention-free directional polling

With contention-free polling, the AP again forms successive directional beams to poll users. Polling messages now include the ID of a user and are addressed to that user. Therefore, the AP needs to know the number but also the IDs of users that are neighbors. The AP attempts to locate each user by sequentially scanning the space with successive directional transmissions. The AP starts by sending a polling message for a user in a beam. If the user does not reside in the beam, the AP does not receive any response and proceeds to formation of the next beam to locate the user. If the user is found to reside in a beam, it responds by a P_ACK message. Upon receiving P_ACK, the AP finds
its spatial signature and sends an ACK message to the user to inform it that its location is found. It then starts scanning anew the space for another user. The order in which users are sought can be arbitrary.

2.3. Medium access

After user positioning, users can access the medium to transmit data. Users that have data to send at each time slot contend to gain access to the medium and transmit data to the AP. The contention resolution method for medium access is the same as the one described above for user positioning. Now, the AP polls users that have data to send, and users attempt to transmit data in a mini-slot with probability $p$. In the sequel, we formulate and solve the problem of minimizing the delay for user positioning. The formulation and solution for the minimum delay access problem are of similar.

3. User positioning and medium access with directional antennas

When the AP needs to learn the position of a user residing within its broadcast range, it can poll the user by broadcast or directional transmission with contention-free or contention-based polling. Contention-free broadcast polling leads to minimum delay for positioning users. However, when the user is out of broadcast range, it cannot be reached by broadcast transmission, and the AP needs to focus transmit power to a directional beam to reach the user. Thus, the AP sequentially steers the beam towards different directions, so that the entire horizon is covered, and maximum range polling through successive directed transmissions is achieved. In contention-free polling, the horizon is successively scanned by directional beams until the user is found, and the procedure is repeated for all users. In contention-based polling, the horizon is successively scanned by a directional beam, and contention among users in a beam is resolved before proceeding to the next beam. The absence of contention is the advantage of contention-free over contention-based polling. However, the scanning delay for locating each user separately may be larger than the delay with contention-based polling.

3.1. Contention-based directional polling: problem statement

In contention-based polling, a parameter that decisively affects the total user positioning delay is the beam width of the beam that scans the space. If a large beam width is used, then fewer beams are needed to scan the space, and the required scanning time with successive directed transmissions is less. However, with a large beam width, the number of users that receive the polling message on average is larger (assuming uniform spatial user distribution), and hence the contention resolution within the beam lasts longer. From that point of view, a large beam width does not imply reduction in the delay for locating all users. A similar tradeoff holds for small beam widths.

Furthermore, the contention resolution mechanism is important. In order to better demonstrate our neighbor positioning approach, we adhere to a simple contention resolution mechanism, which is signified by a transmit persistence probability $p$. We would like to understand how this impacts user positioning delay and to choose an optimal value for it. Assume for instance there exist $n$ unresolved users in a beam. The probability that one user transmits in a slot (and therefore is resolved) is $np(1-p)^{n-1}$. This is maximized for $p^* = 1/n$, which depends on the number of unresolved users $n$. Ideally, the AP could instruct users to retransmit with probability $1/n$ so as to improve the chances of successful user resolution. The problem is that the AP is not aware of the number of users in a beam, and therefore it does not know the number of unresolved users at each step of the procedure. Another major challenge is that the AP does not have knowledge about the spatial distribution of users. If the AP knew that, it would act so as to balance contention by choosing larger beam widths and higher persistence probabilities, if the user spatial distribution were sparse.

The problem we address here is that of minimizing the total delay for user positioning with directional contention-based polling by controlling the beam width and the transmit persistence probability.

Fix attention to a beam with $n$ users and let $i \leq n$. Suppose that $i$ users contend for the medium at a mini-slot and that each user transmits with probability $p$. The probability that only one user transmits out of $i$ users and is therefore resolved is $x_i(p) = ip(1-p)^{i-1}$. Clearly, an average of $1/x_i(p)$ mini-slots are required for a successful user resolution. In the contention resolution mechanism, the first user out of $n$ gets resolved in $1/x_1(p)$ mini-slots on average. Then, resolution is repeated with $n - 1$ users, and so on. At the last stage, the last remaining unresolved user is polled and resolved in one mini-slot without contention. The expected number of required mini-slots to resolve all $n$ users is

$$r_n(p) = 1 + \sum_{i=2}^{n} \frac{1}{x_i(p)} = 1 + \sum_{i=2}^{n} \frac{1}{ip(1-p)^{i-1}}. \tag{2}$$

3.1.1. Known user positions

Assume for a moment that the AP knows the angular locations of the $N$ users. The question that arises is the following: Given the user locations, how should these be grouped in successive directional beams and what should be the persistence probability $p$ so that the resulting contention intervals minimize the total delay with which resolved users access the medium and transmit to the AP? This problem is also meaningful when the $N$ users need to transmit data to the AP, and the AP employs successive directional transmissions to regulate contention and receive user data with minimum delay.

If users are divided into disjoint groups of equal size $g$, with $1 \leq g \leq N$, first we compute the minimum expected user resolution delay per beam, measured in number of mini-slots,

$$r^*_g = \min_{0 \leq p \leq 1} \left( 1 + \sum_{i=2}^{g} \frac{1}{x_i(p)} \right). \tag{3}$$

For group size $g$, there exist $\lfloor N/g \rfloor$ beams with $g$ users each, and one beam with the remaining $(N - g\lfloor N/g \rfloor)$ users. The
total delay for resolving users in all beams, measured in mini-slots, is
\[ \bar{D}_s = \left| \frac{N}{g} \right| r_x^* + r_x^* \frac{N}{g} |N/g|, \]
and the delay in time slots is
\[ D_s = \left| \frac{N}{g} \right| r_x^* + \frac{N}{g} \left| \frac{N}{g} \right| \frac{N-r_x^*}{L}. \]

We choose the group size that leads to minimum total delay. The beam width for each group \( G \) will be \( \theta_x = \max_{i \in G} \phi_i - \min_{i \in G} \phi_i \), where \( \phi_i \) is the angular position of user \( i \).

3.2.1. Numerical solution

Let \( \Delta \) be the metric capturing an estimate of the total expected delay for resolving users in all beams, measured in contention resolution delay in a beam of width \( \theta \). The beam width for each group \( G \) will be \( \theta_x = \max_{i \in G} \phi_i - \min_{i \in G} \phi_i \), where \( \phi_i \) is the angular position of user \( i \).

The probability that the first beam in the scan, and the persistence probability \( p_1 \) of users in that beam. The probability that \( k \) of out \( N \) users reside in a beam of width \( \theta \) is
\[ \omega_k(N, \theta) = \left( \frac{N}{k} \right) \left( \frac{\theta}{2\pi} \right)^k \left( 1 - \frac{\theta}{2\pi} \right)^{N-k}. \]

The expected delay to resolve \( k \) users that have transmit persistence probability \( p \) is \( r_x(p) \), and the total expected contention resolution delay in a beam of width \( \theta \) is
\[ D(\theta) = \sum_{k=1}^{N} \omega_k(N, \theta) r_x(p). \]

The AP controller needs to find the beam width \( \theta_x \) of the first beam in the scan, and the persistence probability \( p_1 \) such that a metric capturing an estimate of the total expected resolution delay is minimized. Namely, it solves:
\[ \min_{0,\theta_x \leq 2\pi, \theta_x \leq 1} 2\pi \theta_x \sum_{k=1}^{N} \omega_k(N, \theta_x) r_x(p_1). \]

Let \( q_1 = \theta_x/2\pi \). The problem is written equivalently as
\[ \min_{0,\theta_x \leq 2\pi, \theta_x \leq 1} \frac{1}{q_1} \sum_{k=1}^{N} \omega_k(N, q_1) r_x(p_1). \]

3.2.1. Numerical solution

Define the objective function at the first stage as:
\[ f(p, q) = \frac{1}{q} \sum_{k=1}^{N} \omega_k(N, q) r_x(p) \]
\[ = \frac{1}{q} \sum_{k=1}^{N} q^k (1 - q)^{N-k} a_k(p), \]
with
\[ a_k(p) = \left( \frac{N}{k} \right) r_x(p). \]

First, we fix \( p \) and minimize function \( f(p, q) \) with respect to \( q \). The derivative of \( f(p, q) \) with respect to \( q \) is
\[ \frac{\partial f(p, q)}{\partial q} = \sum_{k=1}^{N} a_k(p) q^{k-2} (1 - q)^{N-k-1} [k - 1 + q(N - 1)]. \]

We seek roots of this expression in \([0, 1]\). First, note that \( \lim_{q \to 0} \frac{\partial f(p, q)}{\partial q} = +\infty \) and \( \lim_{q \to 1} \frac{\partial f(p, q)}{\partial q} = \lim_{q \to 1} \frac{1}{q} \prod_{k=1}^{N} a_k(p) q^{k-2} (1 - q)^{N-k-1} = -\infty \). Thus, for any \( p \), function \( \frac{\partial f(p, q)}{\partial q} \) has an odd number of roots in \([0, 1]\). We seek a subinterval in \([0, 1]\), in which we will apply the bisection method to find a root. Observe that the first term in the summation above in (11) is \( a_k(p) |1 - q|^{N-k} (1 - N) < 0 \), and the \( k \)th term is \( a_k(p) q^{k-2} (N-1) > 0 \). Also, observe that the \( k \)th term in the summation in (11), \( \ell = 2, ..., N - 1 \) becomes zero for \( q = q_{\ell} = \frac{1}{N - \ell} \) and is negative for \( q > q_{\ell} \).

Also, observe that if the \( k \)th term is negative, then all previous terms \( \ell = 2, ..., \ell - 1 \) are negative as well. This suggests the following algorithm for finding a root of \( \frac{\partial f(p, q)}{\partial q} \) with respect to \( q \):

- **Step 0:** Start with \( \ell = N - 1 \). Set \( q_{N - 1} = \frac{1}{N - 1} \).
- **Step 1:** If \( \frac{\partial f(p, q)}{\partial q} \bigg|_{q = q_{\ell}} > 0 \), search for a root \( q_{\ell} \) of \( \frac{\partial f(p, q)}{\partial q} \) in \([q_{\ell}, 1]\).
- **Step 2:** If such a root \( q_{\ell} \) is found, check if \( \frac{\partial f(p, q)}{\partial q} \bigg|_{q = q_{\ell}} > 0 \), so that \( q_{\ell} \) is local minimum.
- **Step 3:** Continue with checking all \( \ell = N - 1, N - 2, ..., 2 \), by setting \( q_{G} = \frac{1}{N - \ell} \) and searching for a local minimum \( q_{\ell} \) in \([q_{\ell}, 1]\).
- **Step 4:** If more than one value of \( q_{\ell} \) is found, choose for local minimum \( q^* \) the one for which \( f(p, q_{\ell}) \) is minimum.

Note that at each step, the search interval \([q_{\ell}, 1]\) becomes larger.

We now fix the value of \( q \) and seek to minimize function \( f(p, q) \) with respect to \( p \). We have
\[ \frac{\partial f(p, q)}{\partial p} = -N \sum_{k=1}^{N} \omega_k(N, q) \left( \frac{k}{q} - \frac{1}{p} \right)^{N-k} \left( \frac{1}{p^2} (1 - p) \right)^{k-1}. \]

Since \( \lim_{p \to 0} \frac{\partial f(p, q)}{\partial p} = -\infty \) and \( \lim_{p \to 1} \frac{\partial f(p, q)}{\partial p} = \infty \), there exists an odd number of roots of \( p \) in \([0, 1]\). The \( k \)th term in the outer summation in (12) is positive if \( p > 1/k \). Also, observe that, if the \( k \)th term is positive, then all previous terms \( k' = 2, ..., k - 1 \) are positive as well. This suggests the following algorithm for finding a root of \( \frac{\partial f(p, q)}{\partial p} \) with respect to \( p \):

- **Step 0:** Start with \( k = N \). Set \( p_N = 1/N \), so that \( \frac{\partial f(p, q)}{\partial p} > 0 \).
- **Step 1:** If \( \frac{\partial f(p, q)}{\partial p} \bigg|_{p = p_N} > 0 \), search for a root \( p_N^* \) of \( \frac{\partial f(p, q)}{\partial p} \) in \([0, p_N]\).
- **Step 2:** If such a root \( p_N^* \) is found, check if \( \frac{\partial f(p, q)}{\partial p} \bigg|_{p = p_N^*} > 0 \), so that \( p_N^* \) is local minimum.
Step 3: Continue with checking all \( k = N, N - 1, \ldots, 2 \), by setting \( p_k = \frac{1}{k} \) and searching for a local minimum \( p_k^* \) in \([0, p_k] \).

Step 4: If more than one value of \( p_k^* \) is found, choose for local minimum \( p^* \) the one for which \( f(p_k^*, q) \) is minimum.

Again the search interval \([0, p_k] \) increases at each step. In order to find (at least) a local optimum pair of beam width and persistence probability, the AP may iterate between the following problems:

\[
\begin{align*}
(1) & \quad q^{(k)} = \arg\min_{0 < q < 2\pi / h_k} f[p^{(k-1)}, q], \\
(2) & \quad p^{(k)} = \arg\min_{0 < p < 1} f[p, q^{(k)}],
\end{align*}
\]

until convergence to a local minimum \((q^*, p^*)\). Thus, the AP chooses the beam width \( \theta_k = 2\pi q^* \) to form the first beam of the scan. During the control interval of the next time slot, it instructs users in that beam through a control message to use persistence probability \( p_1 = p^* \).

3.2.2. Algorithm

Now, suppose that \( N_1 \) users have been resolved in the first beam with beam width \( \theta_1 \). In order to compute the next beam width \( \theta_2 \) and persistence probability \( p_2 \), the AP should use the fact that \( N_1 \) users have already been resolved and adjust the anticipated amount of contention in the second beam. Assuming again uniform location distribution for users yet to be resolved, the AP controller solves the following problem:

\[
\min_{0 < \theta_2 < 2\pi / \theta_1} \left\{ \frac{2\pi - \theta_1}{\theta_2} \sum_{k=1}^{N} \frac{N_1 - N_k}{N} r_k(p_2) \right\}
\]

and finds a solution \((\theta_2, p_2)\) with the numerical technique outlined above.

In a similar fashion, the AP applies the procedure above at each step and ultimately computes a finite sequence of beam widths \( \{\theta_i\}_{i=1,2, \ldots} \) and respective persistence probabilities \( \{p_i\}_{i=1,2, \ldots} \), where \( \theta_i \) and \( p_i \) are beam width and persistence probability in the \( i \)th beam in the scanning sequence.

Alternatively, the AP may start with an initial beam width \( \theta_1 \) and persistence probability, and after resolving \( N_1 \) users, it can estimate the total number of users as \( \hat{N} = (2\pi / \theta_1) N_1 \) and use it to compute the second beam width and persistence probability. The AP may update its estimate of number of neighbors at each stage based on the number of resolved users up to that stage. If the number of neighbors is not known, the AP may assume \( N \) to be a known upper bound to that number.

3.3. Contention-free directional polling

We now find the expected delay for positioning users with contention-free polling. Consider the case when \( \left\lfloor \frac{2\pi}{\theta} \right\rfloor \) beams of beam width \( \theta \) are formed. The probability that a particular user lies in a beam \( k = 1, \ldots, \left\lfloor \frac{2\pi}{\theta} \right\rfloor \) is \( \frac{\theta}{2\pi} \). The probability that the first beam in the scanning sequence is beam \( \ell, \ell = 1, \ldots, \left\lfloor \frac{2\pi}{\theta} \right\rfloor \) is also \( \frac{\theta}{2\pi} \). Assuming that scanning is performed clockwise, the delay for finding a user that lies in beam \( k \) given that the scanning starts with beam \( \ell \) is

\[
D_{k\ell} = \left\{ \begin{array}{ll}
(k - \ell + 1) & k \geq \ell, \\
\left\lfloor \frac{2\pi}{\theta} \right\rfloor + k - \ell + 1 & k < \ell,
\end{array} \right.
\]

and the total average delay to position all users is

\[
\left( \frac{2}{2\pi} \right) \sum_{k=1}^{\left\lfloor 2\pi/\theta \right\rfloor} \sum_{\ell=1}^{\left\lfloor 2\pi/\theta \right\rfloor} D_{k\ell} = \frac{1}{2} \left( 1 + \left\lfloor \frac{2\pi}{\theta} \right\rfloor \right).
\]

4. Numerical results

In order to demonstrate our approach and to bring into the picture the practical issue that users reside in or out of broadcast range of the AP, we define the following simulation setups. Performance evaluation is all cases was done by using MATLAB.

4.1. Simulation setup 1

First, we consider a scenario of fixed beam width and persistence probability. The area around the AP is covered by \( B \) beams. We consider \( N \) users uniformly distributed in an area around an AP, so that they can reside either in or out of its broadcast range. The AP needs to find the spatial signature of all users and can poll users with omni-directional or directional transmission. At each time, only one beam can be formed towards a certain direction. For users within the broadcast range, the AP may select to poll users by broadcasting or directional beamforming and can use contention-based or contention-free transmission. For users out of broadcast range, the AP can use only beamforming to poll users. The time lengths of the polling, P_ACK and ACK messages, denoted by \( X_p, X_{p,a} \) and \( X_a \) respectively, were chosen to satisfy ratios \( X_p : X_{p,a} : X_a = 1 : 2 : 1 \). This selection is justified by the fact that all messages have an address field and the P_ACK message also has a preamble for spatial signature acquisition. Each contention resolution interval (CRI) consists of \( L \) slots.

4.1.1. Contention-based polling

Let us compute the expected time \( d(n) \) to obtain the spatial signature of \( n \) users in a beam. Define \( s_i,n \) as the probability that \( i \) out of \( n \) users have already been resolved successfully in \( L \) contention resolution slots. Then, \( s_{1,n,1} = w = n(p^{1-p})^{n-1} \) and \( s_{i,n,L} = 0 \), if \( i > n \) or \( i > L \). For all other cases, \( s_{i,n,L} \) can be computed through the recursive equation,

\[
s_{i,n,L} = ws_{i-1,n-1,L-1} + (1 - w)s_{i,n,L-1}.
\]

For the time delay \( d(n) \), we have that \( d(0) = X_p \) and \( d(1) = X_p + X_{p,a} + X_a \), while for \( n \geq 2 \) it is

\[
d(n) = \sum_{k=0}^{n} s_{k,n}[X_p + X_{p,a} + L X_{p,a} + k X_a + d(n - k)].
\]

In the beginning of a CRI, a polling message is sent and the polling response from users (collided or not) is received. If \( k \) out of \( n \) users are resolved in \( L \) contention slots, this means that the AP has received P_ACK messages from
users that retransmit in the intervals following each one of the $L$ contention slots, and it has sent $k$ ACKs to resolved users. The term $d(n - k)$ accounts for the fact that $n - k$ users still need to be resolved.

We now compute in recursive fashion the expected time $D(N, B)$ to resolve all $N$ users and obtain their spatial signatures when the horizon is covered by $B$ successive directed transmissions. Let $\omega_i(N, B)$, given by (6) denote the probability that $i$ out of $N$ users reside in a beam when there are $B$ beams of beam width $2\pi/B$ each. The delay $D(N, B)$ can be computed recursively as

$$D(N, B) = \sum_{i=0}^{N} \omega_i(N, \frac{2\pi}{B})[d(i) + D(N - i, B - 1)], \quad (18)$$

where the first term in the brackets denotes the delay to resolve $i$ users in a beam, and the second term indicates that $N - i$ users need to be resolved in the remaining $B - 1$ beams.

4.1.2. Contention-free polling

In contention-free polling, the delay to locate all $N$ users is highly dependent on the distribution of users in the area around the AP. Clearly, the minimum time delay is achieved when all users reside in the first beam that is formed while scanning the space, while the maximum time delay occurs if all users reside in the beam that is formed last. The expected delay $D'(N, B)$ to obtain the spatial signatures of $N$ users when covering the horizon with $B$ beams is,

$$D'(N, B) = N\left(\frac{B + 1}{2}X_p + X_p X_X + X_p X_X\right). \quad (19)$$

Indeed, for each user the AP issues $(B + 1)/2$ polls on average, and it receives one polling response when the user is located and sends one ACK to the user.

4.1.3. Comparative results

The performance measure is time delay until spatial signatures of users are acquired. We evaluate and compare the performance of four schemes:

- **Broadcasting/Beamforming (Broad/Beam) scheme.** The AP uses contention-free broadcast polling for users within its broadcast range, and it uses polling with beamforming for users out of range. The latter can be contention-based or contention-free.

- **Beamforming/Beamforming (Beam/Beam) scheme.** The AP uses polling with beamforming for all users regardless if they reside in or out of broadcast range. The polling can again be contention-based or contention-free.

In Figs. 3 and 4, we illustrate the performance of the schemes described above for $N = 20$ users for $B = 5$ and $B = 15$ beams respectively. The time delay is plotted as a function of the number of users that reside out of broadcast range, $N_{\text{out}}$. The delay is measured in reference units, each of which corresponds to the duration of a polling message. A first observation is that the performance of the Beam/Beam contention-based and contention-free schemes is independent of $N_{\text{out}}$, since these schemes treat users that are in and out of range similarly. The time delay for contention-free schemes increases linearly with the number of users as can be seen from (19). For $B = 5$, the Broad/Beam and Beam/Beam contention-free schemes perform better than corresponding contention-based ones. This is because the small value of $B$ results in fast enough contention-free polling, and because beams are wide enough, so that the time latency due to user contention is large. When $N_{\text{out}} < 14$, the Broad/Beam contention-free scheme yields the best performance.

The Beam/Beam contention-free scheme is more preferable when $N_{\text{out}} > 15$, because in the Broad/Beam scheme,
some time is wasted in issuing broadcast polling messages that are not received by users residing out of range. When $B = 15$, the behavior is reversed, namely contention-based schemes incur smaller delay than contention-free ones. The large value of $B$ makes contention-free polling time-consuming, while at the same time user contention within each beam becomes low, since beams are narrow. Broad/Beam contention-based polling yields the smallest delay when $N_{\text{out}} < 17$, while Beam/Beam with contention achieves the best performance in all other cases. From other conducted experiments for $B > 10$, it seems that Broad/Beam with contention should be selected when users out of range represent less than 75–80% of the total number of users. Similar conclusions can be drawn from Fig. 5, where the ratio of number of users out of and within range, $N_{\text{out}}/N_{\text{in}}$, was fixed to $1/2$.

4.2. Simulation setup 2

First, we consider the numerical treatment in (3)–(5) in order to assess the impact of the size of the group of users that contend in a beam, on positioning delay. In Fig. 6, we depict the total user positioning delay, measured in time slots, as a function of number of users per beam, $g$, for different number of contention mini-slots, $L$. The underlying assumption is that the same number of users per beam...
contend, and the beam width of each beam corresponding to group \( G \) of users is 
\[
\max_{i \in G} \phi_i - \min_{i \in G} \phi_i, \text{ where } \phi_i \text{ is the angular position of user } i.
\]
The depicted results correspond to numerical evaluation of (5).

A large number of contention resolution mini-slots favors faster resolution, especially for a larger number of users. The optimal contention resolution probabilities for group sizes \( g = 1, 2, \ldots, 10 \) have values 1, 0.5, 0.4, 0.33, 0.28, 0.25, 0.22, 0.2, 0.18 and 0.17 respectively. Given the value of \( L \) one can deduce the values of the user group size that lead to low positioning delay. For example, for \( L = 15 \), the optimal group size is 6 users per beam. The fluctuations observed in the value of delay for \( L = 5, 10, 15 \) correspond to local minima of (5). It can be seen that the number of contention resolution mini-slots is a decisive design parameter that affects total positioning delay.

We now consider optimizing the positioning delay with respect to the beam width and persistence probability, by applying at each stage the rationale and methodology outlined in Section 3. Assume that we operate the system with beam sectors of fixed beam width \( h = 45^\circ \) and we adapt the persistence probability at each step. Assume \( N = 20 \) users located in successive sectors as follows: 3, 4, 5, 0, 1, 4, 0, 3. These number of users are gradually revealed to the AP as users get resolved at each sector. By using the optimization procedure, we find that the user persistence probabilities for each sector are adapted as follows: \( p_1 = 0.33, p_2 = 0.35, p_3 = 0.38, p_4 = 0.43, p_5 = 0.43, p_6 = 0.44, p_7 = 0.49, p_8 = 0.49 \).

5. Discussion

We addressed the cross-layer design problem of minimum-delay neighbor positioning and medium access in a system with one AP and several neighbors around it. We considered the class of methods with contention-based and contention free directional polling that aim at extending the coverage range of the AP by providing access to users out of broadcast range. We assessed the impact of appropriate selection of beam width and persistence probability on the neighbor positioning delay by formulating and solving an optimization problem.

There exist several directions for future study. In our model, we adhered to a rudimentary contention resolution algorithm which was parametrized by a single parameter, the persistence probability and gave rise to a tractable formulation of the problem of positioning delay minimization. Alternative solutions to manage contention such as the IEEE 802.11 access control protocol would be interesting to consider, in which users could possibly adapt the backoff timer selection or the contention window.

An interesting generalization arises in the case of a network with multiple nodes seeking to position their neighbors. In our model, the AP is by default in transmit mode while neighbors are in listen mode. It would be worthwhile to combine the rationale of scanning methods with probabilistic switching between listen and transmit modes, such that a node discovers its neighbors and it is also discovered as neighbor of others with low latency. Another interesting scenario arises if the AP is equipped with several transceivers. Then, several beams can be formed simultaneously to scan the area around the AP towards different directions, so that the time required to position users will be reduced. A synergy between beams could further improve user positioning delay.

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