Adaptive Resource Allocation in SDMA-based Wireless Broadband Networks with OFDM Signaling

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Abstract—**The increasing popularity of wireless broadband access in local and wide area networks is the main expression of the need for flexible and ubiquitous wireless connectivity. In order to satisfy user resource requirements in the presence of volatility of the wireless medium, sophisticated multiple access and adaptation techniques are required, which alleviate channel impairments and increase system throughput. The use of multiple antennas at the base station allows intra-cell channel reuse by multiple spatially separable users through Space Division Multiple Access (SDMA) and hence enhances cell capacity. However, the employment of antennas in the physical layer raises significant issues in medium access control (MAC) layer. In this paper, we investigate the impact of antenna arrays on MAC layer channel allocation in the context of Orthogonal Frequency Division Multiplexing (OFDM), which is the predominantly proposed signaling scheme for wireless broadband access. We propose an algorithm to allocate channels to users based on their spatial separability properties, while appropriately adjusting beamforming weights and transmission rates for each user in a channel. The unified consideration of such adaptive techniques yields significant throughput benefits.**

I. Introduction

The proliferation of enhanced telecommunication services, such as multimedia, telecommuting, fast internet access and video conferencing has stimulated unprecedented demand for broadband access to information sources of every kind. While data transmission rates are growing rapidly in backbone fiber networks or other wire-line networks with the development of xDSL and cable modem products, increasingly demanding applications and user expectations have emerged, both for indoor and outdoor environments. The need for ubiquitous coverage and connectivity in local loop, local area and wider area networks, in conjunction with the increasing demand for mobility, flexibility and easiness of system deployment have necessitated wireless broadband access.

Given the inherent volatility of the wireless medium, a challenging issue in the design of wireless networks is that of ensuring fulfillment of quality of service (QoS) requirements for users, which is synonymous to achieving a given data rate, signal-to-interference and noise ratio (SINR) level or bit error rate (BER) at the receiver of each user. The ability of a network infrastructure to fulfill such QoS requirements and ultimately enhance system capacity depends drastically on the employment of multiple access and signaling schemes that encompass procedures which span the medium access (MAC) and physical layers. These include sophisticated resource management and reuse, reliable channel estimation, adaptation of transmission parameters and exploitation of system diversity attributes. In particular, adaptive physical layer-based transmission techniques provide the potential to adjust parameters such as transmission power, modulation level, symbol rate or forward error correction coding (FEC) rate, based on channel quality. Moreover, the employment of smart antennas in the transmitter constitutes perhaps the most significant means of capacity increase through Space Division Multiple Access (SDMA).

Orthogonal Frequency Division Multiplexing (OFDM) is the predominantly proposed multiple access and signaling scheme for wireless broadband networks [1]. OFDM is included in the IEEE 802.11a and ETSI HIPERLAN/2 standards for wireless LANs [2],[3], the digital audio/video broadcasting (DAB/DVB) standards in Europe and has also been proposed by IEEE 802.16 working group for fixed broadband wireless access. OFDM is based on the principle of multi-carrier transmission, which originally appeared in the design of high speed digital subscriber line (HDSL) modems under the term discrete multi-tone (DMT) [4]. In OFDM, the wide-band spectrum is divided into multiple orthogonal narrow-band subcarriers. Each user symbol is splitted into subsymbols, each subsymbol modulates a different subcarrier and user subsymbols are transmitted in parallel over these low-rate subcarriers. This transmission method results in reduction of effective symbol rate in the channel, mitigation of inter-symbol interference (ISI) and thus significant improvements in system capacity and attainable data rates. Adaptive modulation techniques in OFDM provide the potential to vary the number of transmitted bits that constitute a user subsymbol in a subcarrier, according to instantaneous subcarrier quality in order to maintain acceptable BER per subcarrier [5]. A large number of allocated bits in a subcarrier increases subcarrier throughput, but requires better subcarrier quality to maintain a fixed BER and hence this should be applied in good quality subcarriers. The problem of bit allocation for throughput maximization becomes meaningful when studied subject to a total power constraint for symbols. Then, power water-filling across subcarriers provides the optimal solution for the single-user case [6]. For the multi-user case, sub-optimal centralized [7] or distributed [8] modulation and power control algorithms have been proposed. In [9], the authors present an iterative subcarrier, bit and power allocation algorithm for minimizing transmitted power. Channel allocation with modulation level and power adaptation for throughput increase was also studied in [10] for generic multiple access schemes with orthogonal channels.

Space Division Multiple Access (SDMA) with adaptive antennas enables intra-cell bandwidth reuse by multiplexing spatially separable users in the same channel with the use of adaptive antenna arrays at the base station. On the up-link, spatial separation of cochannel users can be performed by applying filtering and detection algorithms which exploit the degrees of freedom and diversity at the receiver. Hence, the separation problem can be decomposed into independent optimization problems, one for each user, for computing the filtering weight vectors [11]. On the down-link, user separation is more cumbersome, since adaptation of transmission parameters of a single user affects interference level at all receivers. Since receivers are distributed and are not primarily expected to be equipped with multiple antennas, they cannot exploit reception diversity or perform joint signal detection. Therefore, spatial separation of cochannel users is performed by beamforming at the base station. The beam patterns of users are adapted, so as to maintain an acceptable SINR at each receiver. In [12], the authors propose an iterative algorithm for power control and receiver beamforming for a set of cochannel links. The algorithm converges to a feasible solution, if there exists one and minimizes total transmitted power. The optimal solution for that problem is also demonstrated in [13]. A similar algorithm in the context of OFDM is presented in [14]. Downlink beamforming for power minimization in a single-cell system is studied in [15].

Some heuristic methods to consider the impact of antennas on resource allocation are presented in [16] and [17]. However, adaptive resource allocation in the context of OFDM or other multiple access schemes has predominantly been studied independently from intra-cell user spatial separation. Different users in the same cell are assigned to different channels and transmission parameter adaptation is performed independently for each user, without any consideration of the impact of a particular assignment on other users. In addition, resource reuse is usually based on static cell sectorization and beam switching methods, which do not fully capture potential adaptability of user rate requirements and dynamicity of wireless channel. As a consequence, intra-cell resource reuse is limited, resource utilization is inefficient and system capacity is decreased. However, with an appropriate transmission strategy at the base station, the issues of resource allocation and spatial separability of users can be studied jointly. The extent to which users are spatially separable can be adjusted by transmit beamforming and selective user assignment in channels. Then, an acceptable SINR level is ensured at each receiver, while spatially separable users achieve high total transmission rate, so that system throughput is increased.

In this paper, we study the joint problem of intra-cell resource allocation and transmit beamforming in the context of an OFDM/SDMA system. We propose an algorithm to allocate spatially separable users in the same subcarriers, while appropriately adjusting beam patterns of individual users at the transmitter. We also consider a simplified version of resource allocation and beamforming, when channel reuse is not applicable and incorporate user minimum rate requirements and power constraints. The main goal of our study is to identify the benefits of this integrated approach and motivate further research on more composite versions of the problem. These encompass constrained resource allocation and spatial separation problems, as well as algorithms that would be applicable in other multiple access schemes.

The paper is organized as follows. In section II we provide the transmission and channel models and the assumptions used in our approach. In section III we describe the proposed algorithm of subcarrier allocation and user spatial separation, while in section IV we consider constraints for a simplified version of our problem. In section V we derive separability conditions for the case of two users and in section VI we provide numerical results. Finally section VII concludes our study.

II. SYSTEM MODEL

A. Notational remarks

Vectors are assumed to be column ones and are denoted with boldface letters. Superscripts T and H denote the transpose and complex conjugate transpose of a vector or matrix. The notation $||A||$ refers to the sum of all entries of matrix A, while $|\mathbf{w}|$ denotes the magnitude of vector $\mathbf{w}$. The cardinality of a set $\mathcal{X}$ is shown as $|\mathcal{X}|$. The dominant generalized eigenvector $\mathbf{u}(A, B)$ for matrix pair (A, B) is a normalized eigenvector that corresponds to the largest positive eigenvalue that solves the generalized eigenproblem $A\mathbf{x} = \lambda B\mathbf{x}$. When A, B are symmetric and positive definite, this problem is equivalent to eigenproblem $C\mathbf{y} = \lambda\mathbf{y}$, with $C = L^{-1}A(L^{-1})^H$ and $\mathbf{y} = L^H\mathbf{x}$, where L is a non-singular lower triangular matrix that appears in the Cholesky decomposition of B, $B = LL^H$.

B. Transmitter model

We consider a base station which provides coverage to a cell and serves K users. The base station has a M-element adaptive linear antenna array, capable of separating at most $M' \leq M$ cochannel users. We will assume that all degrees of freedom of the array are exploited, so that $M' = M$. Each one of the K receivers has a single omni-directional antenna. The base station employs OFDM transmission with N subcarriers.

An underlying slotted transmission scheme is assumed. Packetized data arrive from higher layers for transmission over the radio channel and each packet occupies one time slot of duration T_s. A packet consists of S OFDM information symbols and one training symbol that is used for channel estimation. Each OFDM symbol of a user consists of bits which must be assigned to different subcarriers. If $b_{n,k}$ bits of user k are assigned to subcarrier n, the total number of bits per OFDM symbol for k is $\sum_{n=1}^{N} b_{n,k}$. These $b_{n,k}$ bits constitute the n-th subsymbol of user k that modulates subcarrier n. User subsymbols can have different number of bits in subcarriers, by using different modulation levels from a set of available QAM constellations. Assuming that the channel is invariant for a slot duration, subsymbols of all symbols of a user within a packet modulate the same subcarriers. The instantaneous transmission rate R_k (in bits/sec) for user k in a slot is,

$$R_k = \frac{1}{T_s} \cdot S \cdot \left(\sum_{n=1}^{N} b_{n,k} \right). \tag{1}$$

Under SDMA, the base station can transmit up to M subsymbols of M different users in each subcarrier by forming M different beam patterns. The configuration of a K-user OFDM/SDMA
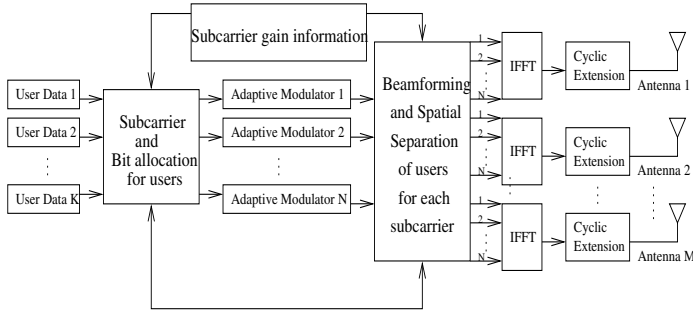
Fig. 1. Block diagram of OFDM/SDMA transmitter with adaptive subcarrier and bit allocation and spatial user separation.

Fig. 2. Beamforming submodule for subcarrier n.

transmitter is depicted in Fig. 1. The serial bit streams of users are inserted into the subcarrier and bit allocation module. This module determines the number of allocated user bits to different subcarriers and the users that share the same subcarriers.

Next, user bits are forwarded into N adaptive modulators. Each modulator modulates the corresponding subcarrier with an appropriate signal that depends on the bits of all users that share the subcarrier. Since linear modulation methods (such as QAM) are assumed, the superposition principle holds and the signal that modulates a subcarrier is given by superimposing the constellation points of users that share the subcarrier. Then, the outputs of modulators enter the beamforming and spatial separation module. For clarity of presentation, the beamforming and spatial separation submodule for subcarrier n is illustrated in Fig. 2. Bits of each of the (at most $M' = M$) users assigned to subcarrier n are forwarded to one of the M parallel beamforming modules. A beamforming vector $\mathbf{w}_{n,k} = \left[w_{n,k}^1, \ldots w_{n,k}^M \right]^T$ is computed for each user k. The output of the beamforming and spatial separation module is a set of beamforming vectors $\{\mathbf{w}_{n,k}\}$, for each subcarrier n and each user k assigned to a subcarrier. The resulting (at most NMM') streams are transformed into time domain samples by inverse fast Fourier transform (IFFT). A cyclic extension of time samples is appended to the signal and serves as guard interval for maximum delay spread. Finally, time samples are transmitted from the M antennas.

C. Channel model

The multi-path channel between the m-th antenna and the k-th receiver is modeled as,

$$h_{m,k}(t) = \sum_{\ell=1}^{L} \beta_k^\ell \, \alpha_m \left(\theta_{\ell,k} \right) \delta \left(t - \tau_k^\ell \right), \quad (2)$$

where L is the total number of paths, β_k^ℓ and τ_k^ℓ are respectively the complex gain and time delay for path ℓ and $\alpha_m \left(\theta_{\ell,k} \right)$ is the m-th element of the $M \times 1$ antenna steering vector at direction $\theta_{\ell,k}$, $\alpha(\theta_{\ell,k})$. The vector $\mathbf{a}_k = \sum_{\ell=1}^{L} \beta_k^\ell \, \alpha(\theta_{\ell,k})$ is called spatial signature of user k and captures spatial properties and the multipath channel of that user.

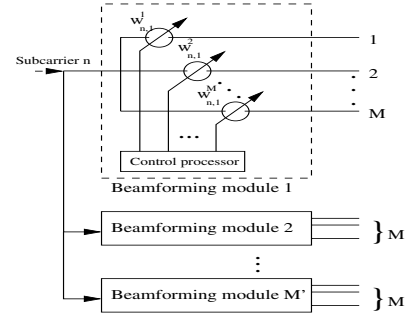The complex baseband OFDM signal of user k that is transmitted from antenna m is,

$$s_{m,k}(t) = \sum_{i=-\infty}^{\infty} \sum_{n=0}^{N-1} w_{n,k}^m \, d_{n,k}^{(i)} \, g \left(t - i(T + T_g) \right) e^{jn(\Delta\omega)t}, \quad (3)$$

where $j = \sqrt{-1}$. The time-domain signal is represented as a sum of rectangular pulses $g(t)$ of duration $T + T_g$, where T and T_g are the symbol and guard interval durations. Each pulse is multiplied by a complex subcarrier coefficient $d_{n,k}^{(i)}$, which denotes the subsymbol of the i-th symbol of user k in subcarrier n. This is shifted in time by i modulation intervals of duration $T + T_g$ and in frequency by $n(\Delta\omega)$, where $\Delta\omega = 2\pi/T$. Assuming that there is no overlap between pulses from different modulation intervals (i.e, different OFDM symbols), we focus on one OFDM symbol, say $i = 0$. It can also be assumed without loss of generality that $T_g = 0$ and that $g(t)$ is normalized to 1. Then, the OFDM symbol of user k that is transmitted from antenna m is,

$$s_{m,k}^0(t) = \sum_{n=0}^{N-1} w_{n,k}^m \, d_{n,k} e^{jn(\Delta\omega)t}. \quad (4)$$

Let $\mathbf{h}_k(t) = [h_{1,k}(t), \ldots, h_{M,k}(t)]^T$ be the channel vector for user k. The signal at receiver k is,

$$y_k(t) = \left(\sum_{j=1}^{K} \mathbf{s}_j^0(t) \right) * \mathbf{h}_k(t) + \nu(t), \quad (5)$$

where $\mathbf{s}_k^0(t) = [s_{1,k}^0(t), \ldots, s_{M,k}^0(t)]^T$ is the transmitted signal of user k from all antennas, $*$ denotes convolution and $\nu(t)$ is thermal noise process at the input of the receiver. Let $\mathbf{H}_{n,k}$ denote the FFT of $\mathbf{h}_k$, evaluated at the n-th subcarrier frequency. This complex quantity captures channel quality in the flat fading environment of a subcarrier. By using the convolution-product property of Fourier transform, we find the received signal of receiver k at subcarrier n as,

$$Y_{n,k} = \mathbf{w}_{n,k}^H \mathbf{H}_{n,k} \, d_{n,k} + \sum_{j=1,j\neq k}^{K} \mathbf{w}_{n,j}^H \mathbf{H}_{n,k} \, d_{n,j} + \nu_{n,k}, \quad (6)$$

where the first two terms are the desired and undesired signals for receiver k and $\nu_{n,k}$ is white Gaussian noise with variance σ^2. Assuming that coefficients $d_{n,k}$ are normalized, the signal-to-interference and noise ratio (SINR) at the output of the matched

filter at subcarrier n of receiver k is,

$$SINR_{n,k} = \frac{\left(\mathbf{w}_{n,k}^H \mathbf{H}_{n,k}\right) \left(\mathbf{w}_{n,k}^H \mathbf{H}_{n,k}\right)^H}{\sum_{j \neq k} \left(\mathbf{w}_{n,j}^H \mathbf{H}_{n,k}\right) \left(\mathbf{w}_{n,j}^H \mathbf{H}_{n,k}\right)^H + \sigma^2} . \quad (7)$$

Define now the $M \times M$ matrix $\mathcal{H}_{n,k} = \mathbf{H}_{n,k}\mathbf{H}_{n,k}^H$. This matrix captures spatial and temporal characteristics of the link between the base station and receiver k at subcarrier n and will be referred to as spatial covariance matrix. Then,

$$SIR_{n,k} = \frac{\mathbf{w}_{n,k}^H \mathcal{H}_{n,k} \mathbf{w}_{n,k}}{\sum_{j \neq k} \mathbf{w}_{n,j}^H \mathcal{H}_{n,k} \mathbf{w}_{n,j} + \sigma^2}. \quad (8)$$

Link quality for all users, subcarriers and antennas is assumed to be known at the transmitter, which means that matrices $\mathcal{H}_{n,k}$ are available for all n, k. With a bidirectional link and time division duplexing (TDD), each subcarrier can be considered as a reciprocal link. Then, spatial signatures of users at each subcarrier remain unchanged for one up-link and down-link transmission. Link quality estimation can be performed at the base station with training symbols that are included in up-link user packets. A user k sends training symbol e_k, consisting of known complex subcarrier coefficients $\{E_{n,k}\}$, $n = 1, \ldots, N$. Antenna m receives N time samples $\{z_{n,k}^m\}$, $n = 1, \ldots, N$ and uses FFT to transform them to frequency domain coefficients $\{Z_{n,k}^m\}$, $n = 1, \ldots, N$. The received coefficient at antenna m and subcarrier n from user k is $Z_{n,k}^m = E_{n,k}H_{n,k}^m + N_{n,k}$, where $N_{n,k}^m$ is the noise component. Then, the MMSE estimate of complex gain is $\hat{H}_{n,k}^m = Z_{n,k}^m / E_{n,k}$. Under the assumption of reciprocity and reasonable time-variance of the link, the transmitter can use these estimates in down-link computations.

The BER of a user that uses M-QAM modulation in a subcarrier is approximated as [18],

$$BER \approx 0.2 e^{-1.5\frac{SINR}{M-1}}. \quad (9)$$

The minimum SINR to maintain BER $\leq q$ for M-QAM modulation is the following SINR threshold,

$$\gamma_M = -\frac{\ln(5q)}{1.5}(M - 1) \quad (10)$$

A higher SINR (lower interference level) in a subcarrier enables the utilization of high M-QAM modulation levels and yields higher transmission rate.

III. RESOURCE ALLOCATION AND BEAMFORMING FOR OFDM/SDMA: THE RESOURCE REUSE CASE

A. Problem Statement

SDMA enables intra-cell reuse of a conventional channel (frequency, time slot, or code) by multiple users, by exploiting spatial properties of the users with respect to the base station. In the case of an OFDM/SDMA system, channel reuse pertains to simultaneous utilization of a subcarrier by several users for transmission.

Two or more users are called *spatially separable* in a subcarrier if they share the subcarrier and the SIR requirements at corresponding receivers are satisfied. For a given subcarrier, spatial separability of users is a function of subcarrier spatial covariance matrices of users, which in turn depend on angular and multipath characteristics of users. User separability also depends on the number of bits that constitute user subsymbols, since different M-QAM constellations are associated with different SINR thresholds and hence different amounts of maximum cochannel interference in order to maintain fixed BER. In addition, beamforming vectors of cochannel users affect SINRs of all receivers corresponding to these users. Finally, user separability depends on each individual subcarrier: users that can share one subcarrier, may not be eligible cochannel users in a different subcarrier, or subcarrier reuse may be feasible for different numbers of allocated bits.

Each user in a subcarrier experiences cochannel interference from other users that use this subcarrier. When a larger number of bits (higher subcarrier transmission rate) is assigned to the user subsymbol in a subcarrier, the throughput for that user is increased. If larger numbers of bits are assigned to subcarriers of this user, the user occupies fewer subcarriers to satisfy certain rate requirements. As a result, more users can be accommodated in the system and capacity is increased. However, a larger number of bits for users in a subcarrier render spatial separability more difficult, since maximum sustainable cochannel interference for an acceptable BER is reduced and hence fewer users can reuse the same channel. Non-separable users should in general be assigned to different subcarriers and from that point of view system capacity is not enhanced. On the other hand, with a smaller number of bits per subcarrier (lower subcarrier transmission rate), a user occupies more subcarriers in order to satisfy rate requirements and thus fewer users can be accommodated in the system. However, a smaller number of assigned bits facilitate spatial separability of more users, since users will be less sensitive to cochannel interference. Therefore, smaller number of assigned bits may even increase capacity. In addition, although users may be spatially separable in different subcarriers, total number of allocated bits per subcarrier may be different, due to differences in spatial covariance matrices and beamforming vectors of users in subcarriers.

The existing tradeoff between total number of assigned bits per subcarrier and spatial separability of users affects system throughput. The arising issue is whether there exists a way to perform subcarrier allocation and user spatial separation jointly, so as to maximize total number of assigned bits per subcarrier and ultimately maximize system capacity. This problem is equivalent to identifying cochannel sets of users that yield larger total number of assigned bits for each subcarrier. Ideally, each subcarrier should have a large number of spatially separated users with large number of allocated bits. This is possible if spatial covariance matrices of users and beamforming vectors are such that users do not induce much interference to each other. For instance, spatial separability occurs for users whose spatial signatures are not highly correlated. However, if angular and multi-path characteristics of users are such that spatial separation is not facilitated, then subcarrier reuse may be feasible only for a subset of users with small number of assigned bits in the subcarrier.

For a particular subcarrier, the goal is to assign as many spatially separable users as possible, while enabling corresponding receivers to satisfy a fixed BER per subcarrier with as many bits per subsymbol as possible. This policy will yield high through-

put per subcarrier. The cardinality of the set of spatially separable cochannel users is limited by the number of antennas, M. The identification of the set of cochannel users that achieves maximum subcarrier throughput is a hard optimization problem. First, an appropriate subset of spatially separable users with certain number of assigned bits per subsymbol must be identified. Then, beamforming vectors for these users must be computed, so that SINR at each receiver is above the SINR threshold that corresponds to that number of bits. The problem is that the SINR at a receiver depends on beamforming vectors of all other users, i.e., it depends on the cochannel set itself. The enumeration of all possible user assignments in a subcarrier is of exponential complexity. In addition, even if the cochannel set of users is fixed, the computation of beamforming vectors that lead to acceptable SINRs at receivers is not straightforward. Therefore, it is desirable to design a suboptimal heuristic algorithm to construct cochannel sets of spatially separable users and find beamforming vectors, so that a large total number of assigned bits per subcarrier is supported.

In this paper, we consider the problem of resource allocation and spatial separation with the objective to increase system capacity and we propose a greedy algorithm to achieve this goal. The algorithm is based on sequential insertion of spatially separable users in subcarriers. In order to keep the complexity of the algorithm to a reasonable level, user reassignments among subcarriers are not considered. The allocation algorithm is applied for an OFDM symbol of each user and the resulting allocation can be replicated for all S symbols of a user packet. Allocation adjustments can be performed once in a certain number of packets, based on received training symbols in the up-link. We first consider the situation when subcarrier reuse is allowed. For illustrative reasons, we present our algorithm under the assumption that the major limitation is cochannel interference rather than noise, so that SINR can be approximated by SIR.

B. Proposed Algorithm

The basic idea of the algorithm is to assign each user to an appropriate subcarrier, by evaluating the incurred rate benefit of the allocation. This benefit is simply the increase in total number of bits of user subsymbols in the subcarrier. Cochannel users in a subcarrier must be spatially separable, while SIRs at receivers must be as high as possible, so as to exceed the largest possible SIR threshold, which corresponds to a large number of bits per subsymbol. Since joint computation of beamforming vectors of users to increase all receiver SIRs is not straightforward, we focus on the impact of the allocation to each user separately. If inserted users cause the least interference to other users that are already assigned in the channel and receive the least interference from them, then spatial separability with larger numbers of bits per user subsymbol are possible, since SIRs at receivers are higher.

For each subcarrier n and user k, we construct the ratio of desired power for user k over undesired power, caused to other users by user k. The desired power is the useful signal power that reaches receiver k, while the undesired power is a measure of interference induced by user k to receivers of other users that use the same subcarrier n. In fact, we are interested in computing the maximum value of this ratio, $D_{n,k}$, by appropriately adjusting the

direction of beamforming vector $\mathbf{w}_{n,k}$. Thus, we define,

$$D_{n,k} = \max_{\mathbf{w}_{n,k}} \frac{\mathbf{w}_{n,k}^H \mathcal{H}_{n,k} \mathbf{w}_{n,k}}{\mathbf{w}_{n,k}^H \left(\sum_{i \in \mathcal{U}^n} \mathcal{H}_{n,i} \right) \mathbf{w}_{n,k}} \ , \ \text{s.t } |\mathbf{w}_{n,k}| = c, \quad (11)$$

where c is a constant and $\mathcal{U}^n$ is the set of users that are already assigned to subcarrier n. The vector $\mathbf{w}_{n,k}^*$ that maximizes the ratio above is known to be the generalized eigenvector of matrix pair $\left(\mathcal{H}_{n,k} \, , \, \sum_{i \in \mathcal{U}^n} \mathcal{H}_{n,i} \right)$ and can be computed with the transformation in subsection II.A. A user k should be inserted in subcarrier n if ratio $D_{n,k}$ is maximum. In addition, we take into consideration the impact of insertion of user k on other users that share the subcarrier. For each such user i already assigned in subcarrier n, we compute the following ratios:

$$D_{n,i}^- = \max_{\mathbf{w}_{n,i}} \frac{\mathbf{w}_{n,i}^H \mathcal{H}_{n,i} \mathbf{w}_{n,i}}{\mathbf{w}_{n,i}^H \left(\sum_{j \in \mathcal{U}^n : j \neq i} \mathcal{H}_{n,j} \right) \mathbf{w}_{n,i}} \quad (12)$$

and

$$D_{n,i}^+(k) = \max_{\mathbf{w}_{n,i}} \frac{\mathbf{w}_{n,i}^H \mathcal{H}_{n,i} \mathbf{w}_{n,i}}{\mathbf{w}_{n,i}^H \left(\sum_{j \in \mathcal{U}^n : j \neq i} \mathcal{H}_{n,j} + \mathcal{H}_{n,k} \right) \mathbf{w}_{n,i}} \ , \quad (13)$$

where in both (12) and (13), we assume $|\mathbf{w}_{n,i}| = c$. The quantity $\Delta D_{n,i}^k = D_{n,i}^- - D_{n,i}^+(k)$ denotes the impact of insertion of new user k on user i that is already assigned in subcarrier n. Insertion of user k causes interference to user i and this is shown in the denominator of $D_{n,i}^+(k)$. A new user k is preferable for the subcarrier if it causes the least total interference to users in the subcarrier, so that quantities $\Delta D_{n,i}^k$ are minimum for as many users i as possible, among all subcarriers n. The Interference Preference Factor (IPF), $I_{n,k}$, is defined as,

$$I_{n,k} = \frac{D_{n,k}}{\sum_{i \in U^n} \Delta D_{n,i}^k} \ . \quad (14)$$

In the allocation process, we want to select a user k and assign it to subcarrier n, so that IPF is maximum. The new cochannel set of users is then more likely to be spatially separable.

Let us assume now that user k is tentatively assigned to subcarrier n, according to the criterion above. The beamforming vectors for users in $\mathcal{U}^n$ are then computed as the solutions to appropriate generalized eigenproblems. These beamforming vectors are used to calculate SIR at each receiver. However, it may happen that insertion of new user k decreases SIR at a receiver of a cochannel user in such a way that the number of bits per subsymbol for that user (and thus the corresponding SIR threshold) needs to be reduced in order to maintain acceptable subcarrier BER and spatial separability of all users. The assignment of a user in a subcarrier is beneficial if the total number of user bits for spatially separable users in $\mathcal{U}^n \cup \{k\}$ exceeds that of $\mathcal{U}^n$ before insertion of user k. If the inserted user does not possess desirable spatial or multi-path properties, e.g., it is not sufficiently separated in angle from other users, the resulting total number of user bits in the subcarrier may be reduced. Clearly, the most appropriate allocation pair (n^*, k^*) of user and subcarrier is the one for which the total increase in number of user bits in that subcarrier is maximized. In order to

formalize our arguments, let $b_{n,k}$ denote the number of bits of tentatively inserted user k in subcarrier n. For each user i, already assigned in subcarrier n, let $b_{n,i}^-$ and $b_{n,i}^+(k)$ denote the number of bits for user i before and after insertion of user k. Increase in the number of bits of a user means an increase in rate of that user in the subcarrier. Thus, we define the Rate Increment Factor (RIF) $T_{n,k}$ for user subcarrier n and user k as,

$$T_{n,k} = b_{n,k} + \sum_{i \in \mathcal{U}^n} \left(b_{n,i}^+(k) - b_{n,i}^- \right) . \qquad (15)$$

An efficient allocation of users to subcarriers that leads to spatially separable users and large total number of user bits is one where user insertion in a subcarrier results in least induced or received interference to and from already assigned cochannel users respectively, as well as in maximum increase in total number of user bits. To capture this objective, we define the Assignment Preference Factor (APF) $A_{n,k}$, as

$$A_{n,k} = I_{n,k} T_{n,k} . \qquad (16)$$

Among assignments with the same amount of induced or received interference, we prefer the one that results in larger rate increase. In addition, among assignments with the same rate benefit, we consider the one with the least induced or received interference, in order to improve the situation for future allocations. The preference factors for the first assigned user k in an empty subcarrier n in the absence of cochannel users are computed as follows:

- The IPF is given by the maximum achievable SIR at the receiver, subject to a power constraint for the user subsymbol,

$$I_{n,k} = \max_{\mathbf{w}_{n,k}} \left(\mathbf{w}_{n,k}^H \mathcal{H}_{n,k} \mathbf{w}_{n,k} \right), \quad \text{s.t.} \quad |\mathbf{w}_{n,k}| = c, \qquad (17)$$

  where c is a constant. The solution $\mathbf{w}_{n,k}^*$ is the eigenvector that corresponds to the maximum positive eigenvalue of matrix $\mathcal{H}_{n,k}$.
- The RIF is the maximum number of bits per subsymbol, which is determined by the maximum available modulation level.

The algorithm selects the allocation that yields maximum APF and assigns that user to the corresponding subcarrier. The procedure is repeated until no further user assignments in subcarriers can increase the total number of user bits in a subcarrier, that is until $T_{n,k} < 0$ for all subcarriers n and users k, or until the cardinality of cochannel sets of users assigned to all subcarriers is M. The main steps of the algorithm are outlined as follows:

Algorithm A

- **STEP 1** : Compute spatial covariance matrices $\mathcal{H}_{n,k}$ for all K users and N subcarriers. Initially all subcarriers are included in list $\mathcal{C}$ of candidate subcarriers for allocation.
- **STEP 2** : Select pair (n^*, k^*) of subcarrier and user that yields maximum APF and perform the assignment.
- **STEP 3** : Update APF factors of cochannel users for n^*.
- **STEP 4** : If $|\mathcal{U}^{\tilde{n}}| = M$ or if $T_{\tilde{n},k} < 0$ for a subcarrier $\tilde{n}$ and all users k, remove subcarrier $\tilde{n}$ from list $\mathcal{C}$.
- **STEP 5** : If list $\mathcal{C}$ is not empty, go to Step 2. Otherwise, terminate the algorithm and go to Step 6.

- **STEP 6** : Repeat the same allocation of users to subcarriers for all packet symbols and all user packets until the next adaptation.

The computationally intense part of the algorithm is the computation of dominant generalized eigenvectors for the APF factors. For one user, this involves Cholesky decomposition of a $M \times M$ matrix and calculation of the maximum eigenvalue of an appropriate matrix, as outlined in II.A. Both of these procedures are known to be of complexity $O(M^3)$. For N subcarriers, K users and M antenna elements, the complexity of the algorithm is $O(NKM^4)$, which is not prohibitive, given that the number of antennas that is used in practice is relatively small (less than 8).

IV. Resource Allocation and Beamforming for OFDM/SDMA: The Constrained Case

A. Motivation and problem statement

In the previous section, we considered the problem of resource allocation with beamforming for spatially separable users and proposed an algorithm for spatial separability with large total number of user bits per subcarrier. However, in a realistic scenario additional constraints may come into stage. For instance, the algorithm generates an allocation of spatially separable users to subcarriers, without any guarantees on total achievable rates for individual users. It may happen that users with certain spatial and multi-path properties, (e.g, spatial covariance matrices) achieve high rates, whereas other users do not. In addition, beamforming power constraints were only utilized to facilitate the assignment. Thus, the only limiting factor in user assignment in a subcarrier was the number of antennas and no statement was made on the impact of beamforming power constraints on the assignment.

Each user k should satisfy a minimum rate requirement ρ_k (in bits/sec), which is usually issued by the MAC layer in order to ensure satisfaction of QoS requirements for users and, to some extent, fairness in resource allocation. In addition, in a multi-carrier transmission scheme such as OFDM, symbol transmission across subcarriers is subject to transmission power constraints per subcarrier. These constraints account for transmission limitations, such as the requirement for low peak-to-average power ratio (PAPR) to avoid symbol distortion [19]. Although the incorporation of constraints in our problem formulation approximates realistic scenarios, the complexity may become prohibitive. For instance, in the assignment procedure of the previous section, user reassignments among subcarriers and additional beamforming calculations should be performed, if the aforementioned constraints are present.

In this section, we consider the impact of these constraints in the context of a simplified problem, where subcarrier reuse is not applicable. In this system, beamforming per subcarrier is used as another dimension to improve subcarrier quality. The OFDM/SDMA transmission system is the one in Figure 1, with the difference that there exists only one beamforming module per subcarrier. Spatial covariance matrices $\mathcal{H}_{n,k}$ are known to the transmitter for each subcarrier n and user k. Due to absence of cochannel interference, the system is assumed to be noise-limited. Noise variance is incorporated in matrix $\mathcal{H}_{n,k}$, so that the SNR at subcarrier n of receiver k is $\mathbf{w}_{n,k}^H \mathcal{H}_{n,k} \mathbf{w}_{n,k}$. Each user k is

assigned to some subcarriers, in order to satisfy minimum rate requirement ρ_k. A fixed BER at each subcarrier must also be satisfied. The number of user bits in a subcarrier depends on the user spatial covariance matrix and the beamforming vector. These determine SNR at the receiver and thus the maximum SNR threshold γ that is satisfied. The beamforming vector for user k in subcarrier n is assumed to have fixed power $P_{n,k}$, i.e., $\mathbf{w}_{n,k}^H \mathbf{w}_{n,k} = P_{n,k}$. A beamforming vector is feasible if it satisfies the power constraint. Clearly, assignment of different users to a subcarrier leads to different feasible beamforming vectors that give a certain SNR at receiver. A subcarrier and bit assignment to users is *admissible*, if there exists a family of feasible beamforming vectors, one for each subcarrier, such that minimum SNR constraints at receivers corresponding to the given number of user bits per subcarrier are satisfied and minimum rate requirements for all users are fulfilled.

The identification of the admissible subcarrier and bit assignment to users that yields maximum total user throughput is not straightforward. It involves selection of the user to be assigned to a subcarrier, computation of the number of user bits in the subcarrier, as well as determination of feasible beamforming vectors. Ideally, each subcarrier should be assigned to the user that can support the largest number of bits in that subcarrier. The problem is that this policy may not lead to an admissible allocation, since some users may not satisfy minimum rate requirements. In the sequel, we propose a heuristic method that identifies an admissible allocation that yields high total user throughput.

B. The proposed approach

Fix attention to a subcarrier n and a user k. The problem of maximizing receiver SNR subject to a beamforming vector power constraint is,

$$\max_{\mathbf{w}_{n,k}} \left(\mathbf{w}_{n,k}^H \mathcal{H}_{n,k} \mathbf{w}_{n,k} \right) \text{ s.t. } \mathbf{w}_{n,k}^H \mathbf{w}_{n,k} = P_{n,k} . \quad (18)$$

Since matrix $\mathcal{H}_{n,k}$ is symmetric and invertible, it can decomposed as $\mathcal{H}_{n,k} = U_{n,k} \Lambda_{n,k} U_{n,k}^H$, where $U_{n,k}$ is a unitary matrix, whose columns are the eigenvectors of $\mathcal{H}_{n,k}$ and $\Lambda_{n,k}$ is a diagonal matrix of the corresponding eigenvalues. Thus $SNR_{n,k} = \tilde{\mathbf{w}}_{n,k} \Lambda_{n,k} \tilde{\mathbf{w}}_{n,k}$, or

$$SNR_{n,k} = \sum_{m=1}^{M} \lambda_{n,k}^m \left(\tilde{w}_{n,k}^m \right)^2 , \quad (19)$$

where $\tilde{\mathbf{w}}_{n,k} = U_{n,k}^T \mathbf{w}_{n,k}$ and $\{ \lambda_{n,k}^m \}_{m=1}^{M}$ are the eigenvalues of $\mathcal{H}_{n,k}$. Note that $\tilde{\mathbf{w}}_{n,k}^H \tilde{\mathbf{w}}_{n,k} = P_{n,k}$, since $U_{n,k}$ is unitary. The maximization (19) subject to the power constraint is achieved by vector [20],

$$\tilde{\mathbf{w}}_{n,k}^* = \sqrt{P_{n,k}} \, [0, \ldots, 0, \underbrace{1}_{\mu_{n,k}\text{-elem}}, 0, \ldots, 0] , \quad (20)$$

where $\mu_{n,k} = \arg \max_m \lambda_{n,k}^m$. Thus, the optimal beamforming vector is $\mathbf{w}_{n,k}^* = U_{n,k} \tilde{\mathbf{w}}_{n,k}^* = \sqrt{P_{n,k}} \, \mathbf{u}_{n,k}^{\mu_{n,k}}$, where $\mathbf{u}_{n,k}^{\mu_{n,k}}$ is the eigenvector corresponding to the largest eigenvalue of $\mathcal{H}_{n,k}$ and the maximum SNR value is $P_{n,k} \lambda_{n,k}^{\mu_{n,k}}$.

Consider now the multi-user case. For each user k and subcarrier n, we compute $SNR_{n,k} = P_{n,k} \lambda_{n,k}^{\mu_{n,k}}$. At each subcarrier n, we assign the user k^* that achieves maximum SNR, namely

$$k^*(n) = \arg \max_{k=1,\ldots,K} \left(P_{n,k} \lambda_{n,k}^{\mu_{n,k}} \right) . \quad (21)$$

The total number of bits (total rate) for each user over all subcarriers is then evaluated by using (1). If all users satisfy the minimum rate requirements, then this is the optimal solution, whereas if no user satisfies minimum rate requirements, then an admissible user and bit allocation does not exist. Assume now that after this initial assignment only a subset of users satisfies minimum rate requirements. Let $\mathcal{S}$ and $\mathcal{U}$ denote the sets of "satisfied" and "unsatisfied" users respectively. Then, the rate of unsatisfied users must be increased, so as to approach and exceed minimum rate requirements. This can be achieved by subcarrier exchange (reassignment) between satisfied and unsatisfied users.

Fix attention to a pair of users $k \in \mathcal{S}$ and $\ell \in \mathcal{U}$ that are initially assigned to subcarriers m and n respectively. Let $r_{n,k}$ denote the number of transmitted bits (rate) for a user k when it is assigned to subcarrier n and let τ_k, τ_ℓ denote the total user rates after the initial assignment. A subcarrier exchange between users k and ℓ is *acceptable*, if the satisfied user does not become unsatisfied and if the rate of the unsatisfied user is increased, or in other words if $r_{m,k} - r_{n,k} \leq \tau_k - \rho_k$ and $r_{m,\ell} \geq r_{n,\ell}$. Subcarrier exchange should cause the *least decrease* in total achievable throughput. For the exchange of subcarriers m and n that are initially occupied by users k and ℓ, we define the reassignment rate cost as,

$$C_{m,n} = (r_{m,k} + r_{n,l}) - (r_{m,\ell} + r_{n,k}) , \quad (22)$$

where terms in parentheses denote rates before and after reassignment and $C_{m,n} \geq 0$. Another significant parameter is the associated rate benefit for *unsatisfied* users. Rate benefit is captured by the metric,

$$U_{m,n} = (\rho_\ell - \tau_\ell) - [\rho_\ell - (\tau_\ell - r_{n,\ell} + r_{m,\ell})]^+ , \quad (23)$$

where the two terms denote the marginal rate that is required in order for ℓ to reach minimum rate ρ_ℓ before and after reassignment and $(x)^+ = x$, if $x > 0$, otherwise it is 0. If the unsatisfied user reaches ρ_ℓ with the subcarrier exchange, then the second term is 0. A preferable reassignment is one with a small reassignment rate cost $C_{m,n}$ and a high rate benefit $U_{m,n}$ for unsatisfied users. We define the Exchange Preference Factor (EPF) as,

$$E_{m,n} = \frac{C_{m,n}}{U_{m,n}} . \quad (24)$$

After initial assignment, admissible subcarrier reassignments with minimum EPF are performed, by selecting appropriate subcarriers occupied by a satisfied and an unsatisfied user. EPFs and user rates are updated after each reassignment. This procedure is iterated until either all users satisfy their minimum rate requirements, or no further acceptable subcarrier reassignments exist. Observe that by requiring maximum rate benefit $U_{m,n}$ for unsatisfied users, we reduce the number of algorithm iterations until user minimum rate requirements are satisfied.

V. Special case: Spatial Separability for two users

In this section, we consider the special case of two users and study required conditions for spatial separability within a subcarrier, given the number of bits of each user's subsymbol in the subcarrier. Let b_i, $\mathbf{w}_i$ and $\mathcal{H}_i$ respectively be the number of bits, beamforming vector and spatial covariance matrix for user i, $i = 1, 2$. In order to have a fixed BER at each receiver for b_i user bits, the minimum required SIR is γ_i, $i = 1, 2$, where the mapping $b \mapsto \gamma$ is performed by using (10), with $b = \log_2 M$. Vector $\mathbf{w}_i$ is written as $\mathbf{w}_i = |\mathbf{w}_i| \, \mathbf{u}_i$, where $\mathbf{u}_i$ is a unit vector that specifies the beamforming orientation. Two users with vector of bit numbers (b_1, b_2) are spatially separable if *both* SIRs at corresponding receivers SIR exceed the required thresholds, namely if,

$$SIR_1 = \frac{\mathbf{w}_1^H \mathcal{H}_1 \mathbf{w}_1}{\mathbf{w}_2^H \mathcal{H}_1 \mathbf{w}_2} \geq \gamma_1 \text{ and } SIR_2 = \frac{\mathbf{w}_2^H \mathcal{H}_2 \mathbf{w}_2}{\mathbf{w}_1^H \mathcal{H}_2 \mathbf{w}_1} \geq \gamma_2 . \quad (25)$$

Then, the vector of bit numbers is called achievable. The arising spatial separability issue is whether there exist beamforming vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ so that condition (25) is satisfied. The determination of beamforming vectors is not straightforward, since SIR of each user depends on both beamforming vectors. Observe that the product of SIRs can be written as,

$$SIR_1 \cdot SIR_2 = \frac{\mathbf{w}_1^H \mathcal{H}_1 \mathbf{w}_1}{\mathbf{w}_1^H \mathcal{H}_2 \mathbf{w}_1} \cdot \frac{\mathbf{w}_2^H \mathcal{H}_2 \mathbf{w}_2}{\mathbf{w}_2^H \mathcal{H}_1 \mathbf{w}_2} = z_1(\mathbf{w}_1) \cdot z_2(\mathbf{w}_2). \quad (26)$$

Then,

$$\max_{\mathbf{w}_1, \mathbf{w}_2} (SIR_1 \cdot SIR_2) = \max_{\mathbf{w}_1} z_1 \cdot \max_{\mathbf{w}_2} z_2 . \quad (27)$$

Let z_1^*, z_2^* denote the maximum values of z_1, z_2, which can be determined by finding the dominant generalized eigenvectors, $\mathbf{u}_1 = \mathbf{u}(\mathcal{H}_1, \mathcal{H}_2)$ and $\mathbf{u}_2 = \mathbf{u}(\mathcal{H}_2, \mathcal{H}_1)$. These eigenvectors are unit vectors, since vector magnitudes are cancelled in (26). Then, if $z_1^* z_2^* < \gamma_1 \gamma_2$, the vector of bit numbers (b_1, b_2) is not achievable and users are not separable. However, if $z_1^* z_2^* \geq \gamma_1 \gamma_2$, we cannot claim that individual bit numbers are achievable. In this case, if we know the beamforming orientation vectors $\mathbf{u}_1$ and $\mathbf{u}_2$, we can write the SIR constraints in the form of a linear inequality system, $F\mathbf{x} \geq \mathbf{0}$ as,

$$\begin{pmatrix} (\mathbf{u}_1^H \mathcal{H}_1 \mathbf{u}_1)/\gamma_1 & -\mathbf{u}_2^H \mathcal{H}_1 \mathbf{u}_2 \\ -\mathbf{u}_1^H \mathcal{H}_2 \mathbf{u}_1 & (\mathbf{u}_2^H \mathcal{H}_2 \mathbf{u}_2)/\gamma_2 \end{pmatrix} \begin{pmatrix} |\mathbf{w}_1| \\ |\mathbf{w}_2| \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \end{pmatrix} . \quad (28)$$

If the system has a solution $(x_1, x_2) = (|\mathbf{w}_1|, |\mathbf{w}_2|)$ with $x_i \geq 0$, $i = 1, 2$, then users are spatially separable for the beamforming orientation $(\mathbf{u}_1, \mathbf{u}_2)$ and the beamforming vectors are given by $\mathbf{w}_i = x_i \mathbf{u}_i$, for $i = 1, 2$. Note that this method is cannot be applied for more than two users, since (27) does not hold.

VI. Numerical Results

A. Simulation setup

We consider a single-cell system with 20 users that are uniformly distributed in the cell area. The system uses OFDM transmission with an array of 8 antennas at 5GHz, in a frequency band which is divided into 50 subcarriers. The received power decreases with distance d from the base station as d^{-4}. Links corresponding to different receivers are uncorrelated. For each such

link, multi-path fading is modeled with a two-ray model. The angle of each path is uniformly distributed in $[0, \pi]$. The delay between paths is uniformly distributed in $[0, T]$, where T is the symbol period. The complex gain of each path is an independent log-normal random variable with standard deviation $\sigma = 6$dB, which accounts for shadow fading. The array steering vector for angle θ is given by vector,

$$\alpha(\theta) = \left(1, e^{(j2\pi d \sin\theta/\lambda_w)}, \ldots, e^{(j2\pi(M-1)d\sin\theta/\lambda_w)} \right), \quad (29)$$

where d is the distance between antenna elements and λ_w is the subcarrier wavelength. The steering vector is thus dependent on subcarrier frequency. Using this model, the spatial covariance matrices $\mathcal{H}_{n,k}$ are determined for each user and subcarrier. A slotted transmission scheme based on TDD is assumed, so that reciprocity between up-link and down-link is ensured. Subcarrier quality estimation is performed by means of training symbols that are transmitted in up-link packets and no estimation errors occur. A target BER of 10^{-3} is assumed for all users and subcarriers and the SINR threshold corresponding to a given number of user bits per subsymbol is computed by (10).

B. Comparative results

The objective of the simulations is to study and compare the performance of different adaptive resource allocation and beamforming techniques. We also consider the effects of intelligent subcarrier and bit allocation and beamforming on system performance. For the problem described in section III, where subcarrier reuse is applicable, the following schemes are compared:

- Subcarrier allocation and spatial separation of users. This is the proposed algorithm in section III. It employs combined subcarrier allocation and beamforming, under the criteria of minimal induced and received interference and large number of transmitted user bits. It will be referred to as Algorithm A.

- Random subcarrier allocation (RSA) and spatial separation of users. This algorithm does not apply the criteria above to assign users to subcarriers. Thus, a user is always assigned to an arbitrary subcarrier. Once user assignment takes place, beamforming vectors are computed as usual and number of user bits per subcarrier are determined. This algorithm is referred to as Algorithm A-RSA.

- Subcarrier allocation with no beamforming (NB). This algorithm does not apply beamforming in the criteria for user insertion in a subcarrier. In the absence of beamforming, preference factors, such as $D_{n,k}$ in (11) are given as

$$D_{n,k} = \frac{||\mathcal{H}_{n,k}||}{\sum_{i \in \mathcal{U}^n} ||\mathcal{H}_{n,i}||} . \quad (30)$$

Other preference factors are computed accordingly. This method is referred to as Algorithm A-NB.

The performance criterion is aggregate subcarrier transmission rate, which is proportional to total number of bits of users in the subcarrier. Different number of bits per subsymbol are achieved by employing 6 different M-QAM modulation levels, which yield up to 6 bits per user OFDM subsymbol in a subcarrier. User

rates were computed over all subcarriers of symbols during transmission of $10,000$ data packets, where channel conditions varied randomly between consecutive packets. Results were averaged over 100 such random experiments with different user locations. Figure 3 illustrates the cumulative distribution function of subcarrier rate for the three algorithms above. For a system with 8 antenna elements and 6 modulation levels, the maximum achievable throughput per subcarrier is 48 bits, since at most 8 users can be separated in a subcarrier. Our integrated approach of subcarrier, bit allocation and beamforming has significantly better performance than the other two schemes, whereas algorithm A-NB turns out to provide the lowest total rate per subcarrier. For example, consider a subcarrier throughput value of 20 bits. With algorithm A-NB, only 20% of subcarriers achieve or exceed this rate, whereas with algorithm A this percentage raises to approximately 70%. Based on performance of algorithms A-NB and A-RSA, it is clear that even with random subcarrier allocation, spatial separation based only on adaptive beamforming yields better results than resource allocation schemes that do not include beamforming.

In Figure 4, the performance of the three adaptive schemes is shown as a function of channel quality, i.e, average link SNR, where a link is now a distinct pair of antenna and receiver. Propagation conditions and user positions were calibrated, so as to create different link conditions. Simulation results show that beamforming based on algorithm A-SRA achieves moderate rate, but performance is substantially improved if adaptive subcarrier allocation is incorporated, as in algorithm A. Note that in severe channel conditions (low SNR values), algorithm A is particularly effective in combating interference. For example, for a link SNR of 4dB, the attainable rate for algorithm A is approximately three and four times higher than rates obtained by algorithm A-RSA and A-NB respectively. The performance of these two algorithms improves as channel conditions improve. Figure 5 shows the effect of different number of antennas and modulation levels on the achievable throughput per subcarrier for algorithm A. For m utilized modulation levels, we assume that these are the highest ones, that is, the ones with $6, \ldots, 6 - m + 1$ bits per subsymbol. To ensure a fair comparison of rates, the depicted rate for the cases of 4 antennas is actually the rate for two subcarriers, which belong in two consecutive symbols. Spatial separation of users is shown to be more effective for a larger number of antennas. Simulation results demonstrate that the inclusion of more modulation levels in the transmission scheme can yield significant rate benefits and this improvement becomes more evident as channel conditions deteriorate, since lower modulation levels can aid spatial separation of users in poor channel conditions.

For the problem described in section IV, where subcarrier reuse is not applicable, Figure 6 demonstrates the link improvements incurred by beamforming. Average subcarrier rate results are provided for different modulation levels and different number of antenna elements. A larger number of lower modulation levels leads to efficient "quantization" of channel conditions and achievability of appropriate rates.

VII. Discussion

In this paper, we addressed the problem of resource allocation and beamforming for spatial separation of users, with the objective to increase resource reuse, guarantee user separability and ultimately enhance system capacity. The problem was studied in the context of an OFDM transmission system, which provides a convenient framework for resource allocation and coherent user separation, due to negligible ISI and delay spread. The determination of user and user bit allocation to subcarriers, as well as of the appropriate beamforming vectors that yield large number of transmitted user bits per subcarrier and maintain acceptable BER at the receiver, is a hard optimization problem. In section V, some separability conditions were derived for a simple system with two users and for a given number of user bits, by exploiting the special structure of SIR requirements. Knowledge of beamforming orientations was also assumed, in order to derive acceptable beamforming vectors. It was stated that these conditions are not valid for more than two users. In such a case, the task would be first to consider all possible subsets of users that can share a subcarrier and then consider all possible combinations of number of user bits, in order to find the subset of separable users that yields maximum total number of transmitted bits per subcarrier. The problem is that any claim about spatial separability of a subset of users cannot be proved, since SIR at a receiver depends on beamforming vectors of other users. Even for a fixed assignment of users, the computation of beamforming vectors that maximize receiver SIRs is a computationally intensive non-linear optimization problem. The consideration of constraints renders the problem even more difficult. Therefore some heuristic algorithm must be applied, whose structure should capture desired properties of a good solution and it will provide performance bounds for more general algorithms.

A greedy heuristic algorithm for the unconstrained problem is proposed in section III. Joint consideration of subcarrier and bit allocation together with beamforming for spatial separation of users is shown to increase system throughput. In section IV, we also considered a simplified version of the problem, where subcarrier reuse was not allowed and applied beamforming power constraints and user minimum rate requirements. A heuristic approach to find an admissible allocation with a high total user rate was presented. An interesting topic for investigation would be to devise efficient algorithms for such composite constrained problems.

In our work, we did not consider any limitation on the number of available beamformers, since a separate beam could be formed for each user and subcarrier. However, in a realistic situation, there will be a limited number of beamformers, mainly due to limited infrastructure cost. The assignment algorithm should then perform user allocation both in the frequency (sub-carriers) and in the space (beamformers) dimension. Additional constraints need to be taken into consideration, such as that users cannot be assigned in the same subcarrier and beamformer simultaneously.

Finally, it is worth pointing out the similarity between spatial separation in the context of SDMA and user separation in other multiple access schemes, such as CDMA, which could lead to issues that warrant further investigation. Spatial user signatures and beamforming in SDMA is the analogue of signature sequences and transmission precoding or receiver filtering respectively in CDMA (depending on whether down-link or up-link is considered). Well-studied resource allocation and parameter adaptation methods used in CDMA could constitute the corner-stones for studying analogous problems in SDMA.

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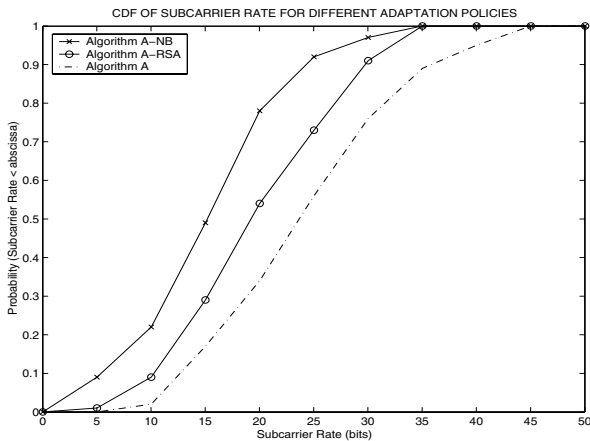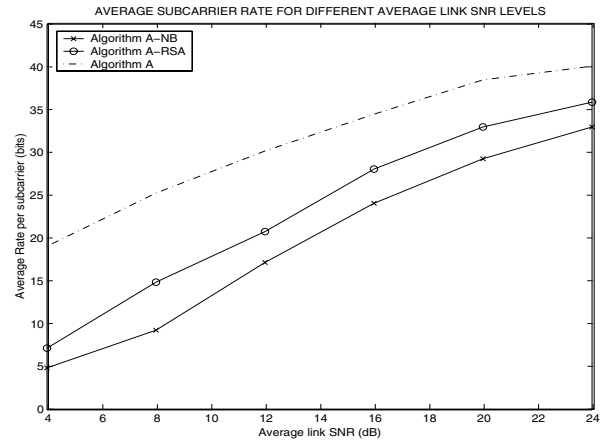[20] D.C. Lay, *Linear Algebra and its applications,* Add.-Wesley, 1997.

Fig. 4. Subcarrier rate for different channel conditions (SNR values).

Fig. 5. Subcarrier rate vs. different number of antennas and modulation levels.

Fig. 3. CDF of subcarrier rate for different adaptive transmission techniques.

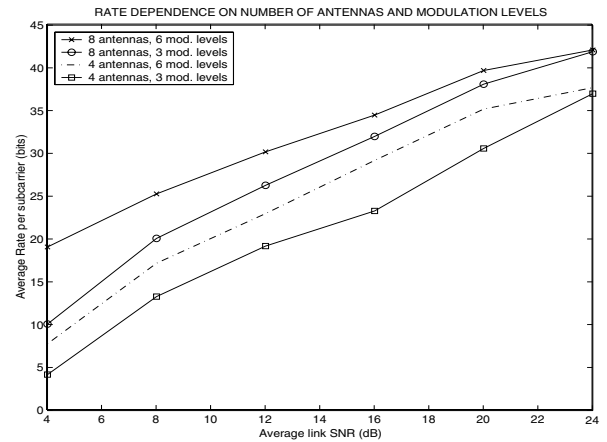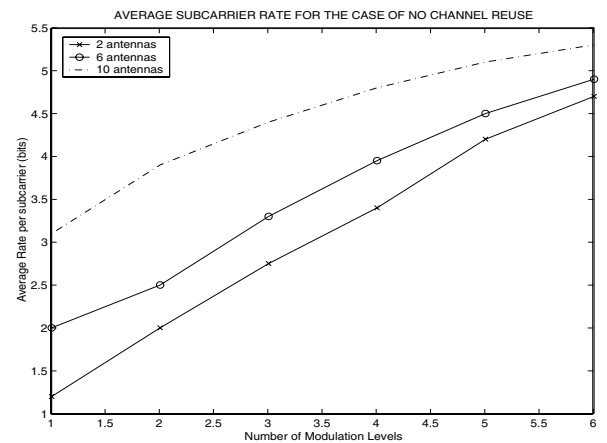Fig. 6. Subcarrier rate vs. different modulation levels for no subcarrier reuse.