The Role of Aggregators in Smart Grid Demand Response Markets

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Abstract—The design of efficient Demand Response (DR) mechanisms for the residential sector entails significant challenges, due to the large number of home users and the negligible impact of each of them on the market. In this paper, we introduce a hierarchical market model for the smart grid where a set of competing aggregators act as intermediaries between the utility operator and the home users. The operator seeks to minimize the smart grid operational cost and offers rewards to aggregators toward this goal. Profit-maximizing aggregators compete to sell DR services to the operator and provide compensation to end-users in order to modify their preferable consumption pattern. Finally, end-users seek to optimize the tradeoff between earnings received from the aggregator and discomfort from having to modify their pattern.

Based on this market model, we first address the benchmark scenario from the point of view of a cost-minimizing operator that has full information about user demands. Then, we consider a DR market, where all entities are self-interested and non-cooperative. The proposed market scheme captures the diverse objectives of the involved entities and, compared to flat pricing, guarantees significant benefits for each. Using realistic demand traces, we quantify the arising DR benefits. Interestingly, users that are extremely willing to modify their consumption pattern do not derive maximum benefit.

Index Terms—aggregator, demand response, electricity market, game theory, optimization theory, smart grid.

I. INTRODUCTION

Recent advances in smart metering technology enable bidirectional communication between the utility operator and the end-users and facilitate the option of dynamic load adaptation. Toward this direction, demand-response (DR) programs provide incentives to major consumers of electricity, usually in the form of monetary rewards, to reduce their electricity consumption in peak-demand periods. DR can take place at a very fast timescale, almost real-time, it leads to a more stable power grid system and it significantly reduces electricity generation cost and CO$_2$ emissions [1].

Although DR has been successfully applied in the industry sector [2], its application in the residential sector is a more challenging task. First, if the existing DR schemes were applied in the residential sector, the operator would derive most of the DR benefits for itself, since each individual amount to a small portion of the total demand and hence has limited negotiation power. Second, the sheer number of home users introduces scalability issues. Third, the utility operator in general lacks the know-how of designing and applying DR mechanisms at such a large scale.

Aggregators are new entities in the electricity market that act as mediators / brokers between users and the utility operator. Aggregators possess the technology to perform DR and are responsible for the installation of the communication and control devices (i.e. smart meters) at end-user premises. Since each aggregator represents a significant amount of total demand in the DR market, it can negotiate on behalf of the home users with the operator more efficiently. The current role of aggregators amounts to paying a monthly fee to the contracted end-users (mainly industrial ones) in order to gain direct control of their appliances [2], [3]. Thus, in case of a peak-demand emergency they can turn off the energy-intensive appliances of the users, e.g. air-conditioning, for a short period.

In this paper, we devise a hierarchical DR market model for the residential sector that captures the interaction of home users, the utility operator, and several competing aggregators. Our model enables efficient demand response in the presence of self-interested entities and structures the DR market in three levels.

At the upper level, the utility operator provides monetary rewards to the aggregators for their DR services and its objective is to minimize its own operational cost. At the middle level, the aggregators provide DR services to the operator by presenting a total demand profile that minimizes the cost of the operator to support it. Instead of direct control of home appliances, we consider the dynamic scenario where the aggregators provide monetary incentives to home users to modify their demand pattern through a day-ahead market. The objective of each aggregator is to maximize its own net profit, namely the income received from the operator minus the compensation it provides to home users.

At the lower level, home users negotiate with the aggregators and receive monetary compensation for their modified consumption pattern. However, such a modification causes inconvenience, since it deviates from preferable or customary usage patterns. Given the day-ahead compensation, the objective of each user is to determine a consumption pattern that strikes an optimal balance of this tradeoff by maximizing a net payoff function. We capture this tradeoff through an inelasticity parameter that is proportional to the inconvenience caused. The market operation that involves all three self-interested entities is summarized in Fig. 1.

The introduction of aggregators in the market introduces novel challenges on how the arising DR benefits span the entire
chain of utility operator, aggregators and end-users. To the best of our knowledge very few recent works have considered the role of the aggregator in the future electricity market e.g. [4], [5]. Our proposed hierarchical market and the corresponding DR mechanism guarantee that no deficit exists in the market and lead to significant benefits for all market participants. Our contributions can be summarized as follows:

- We formulate the objectives of the utility operator, the competing aggregators and the home users in a hierarchical DR market.
- We characterize the operation for the benchmark scenario of a DR market where the operator has full information of all DR-related parameters.
- We devise a DR market scheme, where all entities execute non-cooperative strategies and show the interaction of the entities, when each acts according to its own objective.
- Using realistic demand traces, we show that our hierarchical DR scheme guarantees a portion of the DR benefits for each market entity. Thus, despite the dominant role of the operator, both the aggregators and the end-users can achieve significant financial benefits through negotiation. Interestingly, increasing the willingness of users to modify their consumption pattern (lower inelasticity) reduces the grid operational cost, but is not always beneficial for the aggregators and home users.

Fig. 1. The operation of the proposed day ahead market.

The rest of this paper is organized as follows. In Section II we introduce the hierarchical DR model and formulate the objectives of the involved entities. Section III describes the benchmark scenario of a day-ahead market where all the decision making is performed by the operator under full information about the demands of end-users. In Section IV we present how our model could be applied in a realistic hierarchical market and investigate the required information exchange. In Section V we use daily demand traces to quantify the benefits arising from the proposed market structure. Section VI provides an overview of the related work, while Section VII concludes our study.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider an electricity market consisting of an electric utility operator, a set of aggregators \( A = \{1, 2, \ldots, J\} \) and a set of residential users, as the one depicted in Fig. 2. All the involved entities are self-interested and rational. Each user participates in the market through an a priori determined aggregator on a contractual basis, i.e., users do not move between aggregators. We focus on the day-ahead market and assume that the day is divided into \( T \) equal periods. We denote with \( T = \{1, 2, \ldots, T\} \) the corresponding set of time slots. Throughout the text, we use the terms demand and load interchangeably.

A. The role of the utility operator

In a day-ahead market, the ensemble of users issues a cumulative demand of \( W \) Watts. Under the current practice of flat pricing, a fixed price \( q_f \) is charged per Watt of consumption, independently of the period of the day. Thus, within a day the operator derives a total income of \( q_f W \) from the users payments.

In order to meet demand, the operator has either to activate costly powerplants or to purchase electricity from third parties. The cost of generating electricity power is generally assumed to vary with time, due to the time-varying availability of supply, e.g. from renewable sources. However, for any given time slot \( t \) cost is a strictly increasing and convex function \( c_t(y_t) \) of the corresponding total load \( y_t \geq 0 \) [6].

The operator seeks to calculate the cumulative daily load profile vector \( y = \{y_t : t \in T\} \) that maximizes its revenue.
If the operator could directly control the loads, its objective would be formally expressed as:

$$\max_y \quad q_f W - \sum_{t \in T} c_t(y_t)$$ 
\hspace{1cm} (1)

s.t. \quad \sum_{t \in T} y_t = W.

Since the received income is fixed, (1) can be transformed into a convex cost minimization problem, which can be easily solved through convex optimization. Indicatively, for time-invariant cost functions, the optimal solution is a perfectly balanced load across the day.

Since the operator has no means to exercise direct control over the user demands, alternative means of indirect control such as dynamic pricing have been proposed, e.g. in [7], [8]. The large number of home users and the fact that each of them has negligible impact on the total cost makes such an approach challenging though. This challenge motivates the introduction of the aggregator as an intermediary entity between the operator and the end users.

In our model, the operator provides monetary rewards $\lambda = \{\lambda_j \geq 0 : j \in A\}$ to the aggregators so that they perform DR on its behalf. Note that in this case the load vector $y$ becomes dependent on $\lambda$. In particular, the operator is willing to provide a portion $\hat{\lambda} = \sum_{j \in A} \lambda_j$ of its DR gain to the aggregators. The DR gain is the reduction of the power generation cost that results from reward $\lambda$ and is given by

$$\Delta c(y(\lambda)) = \sum_{t \in T} \Delta c_t(y_t(\lambda)) = \sum_{t \in T} \left[ c_t^0 - c_t(y_t(\lambda)) \right],$$  
\hspace{1cm} (2)

where $c_t^0$ is the power generation cost at timeslot $t$ if no DR is applied.

Thus, in the presence of aggregators, problem (1) becomes:

**Operator’s problem (min operational cost):**

$$\min \lambda \quad \sum_{t \in T} c_t(y_t(\lambda)) + \hat{\lambda} \Delta c(y_t(\lambda))$$ 
\hspace{1cm} (3)

s.t. \quad 0 \leq \hat{\lambda} \leq 1

$$\lambda_j \geq 0 \quad \forall j \in A.$$  

The objective function of the operator captures all its expenses in a DR market, i.e. both power generation cost and its reward to the aggregators for their services. Note that the reward provided to the aggregators depends only on the quality of their aggregate DR services. The exact way that reward is allocated to the aggregators is addressed in Section IV.

**B. The role of the aggregators**

Since home users cannot negotiate directly with the operator, they enroll in a DR program provided by an aggregator that aggregates several small residential DR assets into a larger unit, in order to increase their negotiation power. Given that each user is assigned to an aggregator through a contract, we denote with $D_j$ the set of demands of all the users (i.e. from all individual appliances) under aggregator $j$. The role of the aggregator is twofold: i) to provide DR services to the operator and ii) to guarantee a reduced electricity bill to the end-users.

Each aggregator tries to shape the load pattern of its users and receives compensation for the cost savings incurred to the operator due to this shaping. We assume that an aggregator incentivizes users to modify their power consumption profile through dynamic compensation per unit of power. The strategy of aggregator $j$ constitutes of the compensation vector $p_{jt} = \{p_{jt} : t \in T\}$. Let $d_{jt} = \{d_{jt} : t \in T\}$ denote the cumulative load of aggregator $j$ at time slot $t$, over all the demands in $D_j$, that results from compensation $p_{jt}$.

From the side of aggregator $j$, the DR gain $\Delta c$ of (2) depends on its own compensation strategy $p_j$, but also on the compensation strategy of aggregators other than $j$ denoted by $P_{-j} = \{p_{jt} : t \in T\}$. The same holds also for the actual reward received by aggregator $j$, since power generation cost at time slot $t$ is a function of the corresponding total load $y_t = \sum_{j \in A} d_{jt}$.

The objective of aggregator $j$ is to maximize its net profit by solving the following optimization problem:

**Aggregator’s $j$ problem (max net profit):**

$$\max_{p_j} \quad \lambda_j \Delta c(p_{jt}, P_{-j}) - \sum_{t \in T} p_{jt} d_{jt}(p_j)$$ 
\hspace{1cm} (4)

s.t. \quad p_{jt} \geq 0 \quad \forall t \in T.$$  
\hspace{1cm} (5)

The first term corresponds to the reward received from the operator, while the second term is the compensation provided to the users.

**C. Residential demand scheduling**

At the home premises, under the current model of flat pricing, users tend to use their appliances at the most convenient time throughout the day, driven by their personal preference. For example, most people activate cooling within the hottest period of a day, thus creating demand peaks. We define as $x^0_i = \{x^0_{it} : t \in T\}$ the reference consumption profile of appliance $i$ that reflects its preferable power consumption profile in the absence of any DR incentives.

In our model, monetary compensation provided by the aggregators motivates users to move load out of peak consumption periods. We assume that end-users are price-taking since they control a negligible amount of demand and hence cannot affect the compensation strategy of the aggregator. The users issue a set of demands $D$ for the following day. Each demand corresponds to the daily activation profile of a specific appliance of a user and is characterized by a total daily electricity requirement of $W_i$ Watts. We assume $W_i$ to be fixed and independent of the provided rewards. This enables a fair comparison with other pricing schemes, such as flat pricing, since the assumption of curtailable demands further amplifies the benefits of any DR scheme. The operator charges a flat price $q_f$ for each Watt of consumption, which incurs a fixed daily cost of $q_f W_i$ for demand $i$.

For each demand $i$ we define control $x_i = \{x_{it} : t \in T\}$ which corresponds to the power consumption pattern for the following day. Clearly, for each aggregator $j$ holds $d_{jt} = \sum_{i \in D_j} x_{it}$. Note that although a user generally generates multiple demands within a day, these are not coupled. Thus, for each demand $i \in D_j$ the objective of the corresponding
user is to maximize its net payoff, i.e. the compensation received by the aggregator minus the incurred dissatisfaction:

**User problem (max net payoff):**

\[
\begin{align*}
\max_{x_i} & \quad \sum_{t \in T} [x_{it} p_{jt} - V_i(x_{it})] \\
\text{s.t.} & \quad x_{it} \geq 0, \quad \forall t \in T \\
& \sum_{t \in T} x_{it} = W_i,
\end{align*}
\]  

where the *disutility* function \(V_i(\cdot)\) captures the dissatisfaction caused due to deviation from the reference consumption. Function \(V_i(\cdot)\) may be taken to be convex, since the differential dissatisfaction of a user increases as the amount of deviation from the reference power consumption increases.

An indicative example of such a function that we also use throughout this paper is

\[
V_{it}(x_{it}) = v_i(x_{it} - x_{it}^0)^2.
\]

We call \(v_i \geq 0\) the inelasticity parameter of demand \(i\). This parameter enables us to model the different behaviour of the diverse appliances existing in a house. Small enough values of \(v_i\) indicate demands that cause minimum dissatisfaction if their consumption pattern is modified (e.g. the washing machine). On the other hand, a large inelasticity parameter indicates a sensitive demand that causes significant inconvenience (e.g. cooking).

### III. A BENCHMARK MODEL FOR UTILITY-AGGREGATOR-USER INTERACTION

In this section, we consider a benchmark scenario of full information, where the utility operator has global knowledge of the system parameters, namely the reference power consumption profiles \(x_i\), the inelasticity parameters \(v_i\) and the set of allocated users \(D_j\) to each aggregator \(j\). This scenario captures the case where all the involved entities are willing to report their parameters to the operator.

This approach provides insight on how misaligned are the interests of the market entities and whether disclosing this information to the operator is beneficial for the lower levels. It also serves as a benchmark regarding the cost of the operator. We formulate the problem as a multilevel optimization problem [9] and provide a characterization of the solution for the scenario where i) at the lower level the users modify their demand pattern according to the compensation advertised by the aggregators, ii) at the middle level the aggregators determine their compensation strategy so as to maximize their net profit, given the rewards offered to them by the operator, and iii) at the upper level the operator computes the reward per unit of cost reduction to provide to the aggregators so as to minimize its operational cost.

#### A. Aggregator-user interaction

The aggregator would ideally like to control the user appliances directly, in order to impose the consumption pattern that maximizes its net profit. However, each demand of a user participates in demand response according to its own disutility function. Thus, the aggregator has to provide incentives to its end-users in the form of compensation in order to motivate them to modify their demand load pattern. In particular, for a given reward vector \(\lambda_j\) from the operator, each aggregator has to find its compensation strategy \(p_j\) that solves (4).

The optimal strategy of aggregator \(j\) depends on the aggregate demand profile of its users throughout the day, \(d_j\), which is directly affected by its compensation strategy \(p_j\) through (6). In order to calculate its optimal response, the aggregator needs to know the analytical form of \(d_j(p_j) = \sum_{i \in D_j} x_{it}(p_j)\), which in our scenario is increasing function of the corresponding reward \(p_{jt}\), but it is decreasing in the reward provided for any other slot, i.e. \(\frac{\partial d_j(p_j)}{\partial p_{jt}} \geq 0\) and \(\frac{\partial d_j(p_j)}{\partial p_{jt}} \leq 0 \forall t \neq t\). This is expected since increasing compensation for a specific time slot motivates users to move load there, whereas increasing compensation in any other slot motivates users to move load out of it.

In the full information case, the aggregator is aware of the disutility functions of appliances and hence can calculate from the Langrange optimality conditions the best response for each demand. For a given reward vector \(p_j\), (6) is a convex optimization problem in \(x_i\). In order to derive an analytic expression, we discard (7) and introduce it later through a constraint in the corresponding compensation strategy of the aggregator. The Langrangian of demand \(i\) is then given by:

\[
L(x_{it}, \gamma_i) = \sum_{t \in T} [x_{it} p_{jt} - V_{it}(x_{it})] + \gamma_i \left( \sum_{t \in T} x_{it} - W_i \right),
\]

where \(\gamma_i\) is the Lagrange multiplier corresponding to constraint (8). Then, due to convexity of the objective function, the KKT conditions are necessary and sufficient conditions for optimality. Thus, we may derive the following \(T\) optimality conditions:

\[
\frac{\partial L(x_{it}, \gamma_i)}{\partial x_{it}} = 0 \implies p_{jt}(x_{it}, \gamma_i) = \frac{\partial V_{it}(x_{it})}{\partial x_{it}} - \gamma_i, \quad \forall t \in T.
\]

From (11) we can calculate the demand portion \(x_{it}\) of each demand \(i\) at each time slot \(t\) as an expression of \(p_j\) and \(\gamma_i\). Note that the strict convexity of the disutility function guarantees that its partial derivative is strictly increasing and hence invertible. Thus, for the disutility function in (9) we get:

\[
x_{it}(p_{jt}, \gamma_i) = \frac{p_{jt} + \gamma_i}{2v_i} + x_{it}^0, \quad \forall t \in T.
\]

In addition, the equality constraint (8) needs to hold. Under the condition that none of the constraints (7) is active, by replacing \(x_{it}\) from (12) to (8), the aggregator can compute \(\gamma_i\) as an expression of \(p_j\) as \(\gamma_i(p_j) = -\frac{1}{T} \sum_{t \in T} p_{jt} = -\bar{p}_j\), with \(\bar{p}_j\) denoting the average compensation provided by aggregator \(j\) over the day horizon \(T\). By replacing this into (12), we derive the actual distribution of each demand as a function of the compensation vector:

\[
x_i(p_j) = \frac{1}{2v_i} (p_j - \bar{p}_j 1) + x_i^0,
\]

with \(1\) denoting the column vector of ones of dimension \(T\).

Then, since there is no coupling among different demands, each aggregator \(j\) can derive \(p_j\) by replacing...
\[ d_{jt}(p_j) = \sum_{i \in D_j} x_{it}(p_j) \] into (4), i.e. by solving:

\[
\max_{p_j} \lambda_j \sum_{t=1}^{T} \Delta c_t \left( \sum_{i \in D_j} x_{it}(p_j), P_j \right) - \sum_{t=1}^{T} \sum_{i \in D_j} x_{it}(p_j) \nonumber
\]

\[ \text{s.t. } (5), (7), \forall i \in D_j, t \in T. \]

Note that for each time slot \( t \), \( c_t \) is convex in \( d_{jt} \), which in turn is convex and non-decreasing in \( p_{jt} \). Thus, \( c_t(d_{jt}(p_{jt})) \) is also convex.

**B. Operator-aggregator interaction**

The operator has to find the monetary reward vector \( \lambda^* \) that minimizes its operational cost. However, the exact impact of its rewards on demand distribution is difficult to quantify, since it involves also the optimization problems of the lower two levels. In particular, the operator needs to know the analytical expression of \( d_{jt}(p_j(\lambda_j)) \). Notice that the reward strategy of the operator determines the reward provided by the aggregator, which in turn determines the demands of the users.

This problem falls within the class of multi-level optimization problems, which are particularly challenging to solve [9]. In the previous section, we showed how the lower two levels can be merged into one optimization problem. However, the previously derived solution is a numerical one, while in order to characterize the DR solution from the operator’s point of view, we need an analytic expression for the optimal solution of the problem of each aggregator. Since in general such an expression cannot be derived, we calculate the reward strategy of the operator numerically and evaluate it in Section V.

**IV. A NON-COOPERATIVE MARKET MECHANISM**

In the previous section, we assumed that all the information about end-users (demand load and disutility functions) and about the total demand load under each aggregator is available at the operator side. We used this assumption to focus on the problem from the point of view of the utility operator. Such a scenario of information exchange could be plausible in the case that such exchanges are part of prior agreements between the involved entities.

In this section, we consider how the hierarchical market may function in a setting, where the aforementioned assumption of information exchange is relaxed. We propose a top-down approach where at the higher level the operator announces its reward strategy to the aggregators, while on the lower level each aggregator presents monetary compensations to end-users and negotiates with them about the DR services that they can provide for this compensation. Finally, each aggregator responds to the operator with a cost reduction offer. All the required information exchange takes place during the previous day, forming thus a day-ahead DR market.

Starting from the upper level, the operator has to find the monetary reward vector \( \lambda \) that minimizes its operational cost given by (3). The total gain of the proposed DR market is captured by \( \Delta c \) from (2), which is an increasing function of the provided reward \( \lambda \). Each aggregator receives a portion \( \lambda_j \Delta c \) of this gain and provides part of it to the end-users in order to incentivize them to modify their consumption pattern. On the other hand the operator derives the remaining gain \((1-\lambda)\Delta c\). However, the exact impact of reward vector \( \lambda \) on the operational cost of the operator is unknown, due to the involvement of the aggregators and the private disutility functions of the users. Thus, we propose a repeated auction mechanism with iterative elimination at each iteration step, where the aggregators compete with each other to sell their DR services.

At each iteration \( k \), the operator announces the current total demand load \( y^k = \{y^k_t \geq 0 : t \in T \} \) and a scalar total reward \( \lambda^k \). Then, it conducts a first price sealed-bid auction, where each aggregator announces to the operator the DR service that it can provide for this reward. The operator accepts the bid that guarantees the highest cost reduction, and this aggregator is removed from subsequent auctions. Then, reward per unit of cost reduction is increased by the operator according to \( \lambda^{k+1} = \lambda^k + \xi \), and a new auction is conducted with the remaining aggregators.

By increasing the provided reward, the operator enables the aggregators to provide even higher compensation to the home users. Hence, the users get sufficient incentives to move even less elastic load out of peak-demand periods. This process is repeated until either there are no remaining aggregators, or the provided reward \( \lambda^k \) becomes larger than 1 which violates the constraint of the operator.

In order to calculate its DR bid, the aggregator receives the modified consumption patterns of the users in response to the advertised compensation. Each aggregator would like to provide the minimum compensation to end-users, in order to reap maximum benefit for itself. However, such a strategy would reduce the quality of its DR services to the operator and consequently it would lower the chances that its bid wins over the bids of other aggregators. Thus, the competition of the aggregators is implicitly beneficial for the end-users.

Note also that the seemingly plausible strategy of withdrawing from the first auction rounds does not guarantee increased benefit for the aggregator. On the one hand, \( \lambda^k \), the operator reward per unit of generation cost reduction, increases across iterations. On the other hand, each accepted bid leads to a more balanced total consumption pattern. Thus, later iterations provide only limited opportunities for further balancing, which in turn may lead to reduced profit for the aggregator despite the increasing reward.

At the lower level, the aggregator needs to know for each time slot \( t \) the analytical form of \( d_{jt}(p_j) \), which by (11) is a function of \( \frac{\partial V_{ut}(x_{ut})}{\partial x_{ut}} \), so as to calculate its optimal bid. Since the aggregator is unaware of the user disutility functions, such information may only be deduced by observing user responses to the announced compensation. For the single parameter disutility function (9) and under the assumption that the aggregator is aware of the generic form of the disutility function, it can deduce the inelasticity parameter through the following estimation phase. Each aggregator \( j \) selects an arbitrary \( \ell \in T \) and announces a unit compensation vector \( p^j_\ell \), where \( p_{jt} = 1 \) and \( p_{jt} = 0, \forall t \in T \setminus \{\ell\} \). Through the modified load pattern \( x^t_j(p^j_\ell) \), the aggregator can deduce the
exact form of the utility function for each demand $i$, since it can calculate the corresponding $v_i$ parameters from (13). Note that this operation has to be performed only once.

At each iteration $k$, each aggregator $j$ receives the total consumption vector $y_{k}^{h}$ and reward $\lambda_{k}^{j}$ from the operator. Thus, aggregator $j$ can replace $d_{jk}^{h}(p_{j})$ into (4) and solve it similarly to the full information case. Note that the total load of aggregators other than $j$ can be calculated as $d_{k}^{e,j} = y_{k}^{h} - d_{jk}^{h}$.

The exact operation of the proposed day-ahead market is described in detail in Algorithm 1. For notational simplicity we omit the function arguments.

Algorithm 1 The operation of the day-ahead DR market

<table>
<thead>
<tr>
<th>Input: $\xi$: auction price increment step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: $\lambda^{0} \leftarrow 0$ // operator’s zero reward</td>
</tr>
<tr>
<td>2: $p_{j}^{0} \leftarrow 0$ // aggregators’ zero compensation</td>
</tr>
<tr>
<td>3: $\hat{v}<em>{i}^{0} \leftarrow \sum</em>{t \in T} c_{t}(y_{k}^{0})$ // demand under flat pricing</td>
</tr>
<tr>
<td>4: for all $j \in A$ do // estimation of inelasticity param. $v_i$</td>
</tr>
<tr>
<td>5: aggregator announces unit reward vector $p_{j}^{f}$</td>
</tr>
<tr>
<td>6: for each demand $i$ corresponding user responds by $x_{i}$</td>
</tr>
<tr>
<td>7: aggregator calculates $v_{i}$ from (13)</td>
</tr>
<tr>
<td>8: end for</td>
</tr>
<tr>
<td>9: repeat // iteration $k$</td>
</tr>
<tr>
<td>10: operator announces total demand vector $y_{k}^{k-1}$</td>
</tr>
<tr>
<td>11: $\lambda_{k}^{f} \leftarrow \lambda_{k-1}^{f} + \xi$ // the operator increases reward by $\xi &gt; 0$</td>
</tr>
<tr>
<td>12: for all $j \in A$ do // each agg calculates its response bid</td>
</tr>
<tr>
<td>13: aggregator $j$ calculates compensation $p_{j}^{c}$ from (14)</td>
</tr>
<tr>
<td>14: aggregator $j$ responds by $d_{jk}^{h}$ to operator</td>
</tr>
<tr>
<td>15: end for</td>
</tr>
<tr>
<td>16: operator accepts the maximum bid</td>
</tr>
<tr>
<td>17: $A \leftarrow A \setminus {j}$ // accepted aggregator $j$ is removed from negotiations</td>
</tr>
<tr>
<td>18: until $\lambda_{k}^{f} &lt; 1 - \epsilon$ // convergence check</td>
</tr>
</tbody>
</table>

Note that lines 1 – 3 describe a required initialization phase where the aggregators and the operator deduce the behaviour of users under flat pricing. In this direction, the operator announces zero reward to the aggregators for the following day, in order to deduce the reference consumption pattern of the residential users $x_{i}^{0}$. Given the zero reward from the operator to the aggregators, the latter have no incentive to provide any DR services to the operator and hence announce zero compensation to the users. The end-users respond with the reference consumption pattern that they prefer in the absence of incentives. Lines 4 – 8 correspond to the inelasticity estimation phase, where each aggregator calculates the inelasticity parameters of the demands (appliances) of the users that are allocated to it. Lines 9 – 18 constitute the main body of the proposed market, where the operator increases the provided reward, while the aggregators compete with each other through their bids.

Remark 1: Note that a nonprofit operator that only cares for the total benefit of the system would set $\hat{\lambda} = 1$. This would maximize the DR market gain $\Delta c$ but it would also lead to operational cost $c^{0}$, which equals to that of flat pricing (i.e. no DR).

Remark 2: Given that the flat price $q_{f}$ is set so as to cover operator expenses, i.e. $q_{f} \sum_{i} W_{i} = c^{0}$ the proposed market has no deficit, since for any value of $\hat{\lambda}$ always holds $c(\hat{\lambda}) \leq c^{0}$.

V. Numerical Evaluation

In order to evaluate the performance of the proposed market, we rely on a realistic DR dataset based on the per appliance demand generator of [10]. Our dataset captures the daily schedule of 33 types of appliances of diverse characteristics that may exist in a residence. We assume that the disutility of each demand (appliance) is given by the parametric disutility function (9) with $v_{i}$ uniformly distributed in $[0, v_{max}]$, with $v = v_{max}/2$ denoting the mean inelasticity value.

We consider a market of a single operator, $J$ aggregators and $N = 10^{4}$ households equally divided among the aggregators. In order to quantify the benefits of our DR scheme, we use the power generation cost function $c(y_{t}) = y_{t}^{2}$, which is a convex and increasing function of the corresponding total demand $y_{t}$ at each timeslot $t$.

Fig. 3 and Fig. 4 depict the total consumption pattern across a day for the market approach of Section IV for scenarios of low and high mean inelasticity $v$ respectively. Fig. 3(a) and Fig. 4(a) correspond to the default case of flat pricing where no compensation is provided to the users. For the case of low inelasticity, Fig. 3(b) and Fig. 3(c) show the evolution of the load pattern across a day as the DR bids of the aggregators are accepted. The introduction of the aggregators’ DR services smooths the load pattern significantly. In the end, the total demand is spread throughout the day. Instead, in the scenario of higher inelasticity depicted in Fig. 4(b) and Fig. 4(c), consumption is less evenly distributed across the day. Thus, inelasticity of demands, captured by inelasticity parameter $v_{i}$, plays a central role on whether the resulting consumption load pattern will be smooth across the day.

Next, we quantify the DR benefits for each of the participating entities, namely the net payoff derived by the end-users given by (6), the net benefit of the aggregators given by (4) and the total cost of the operator captured by (3). Note that the actual reduction of the user’s electricity bill is at least as much his net payoff, since the latter contains also the disutility term. Therefore, our findings in terms of net payoff can be directly translated to monetary savings in the electricity bill of the user. For comparison, remember that under flat pricing the cost of the operator is $c^{0}$, while all the aggregators and the users derive zero payoff.

First, we consider the impact of the reward strategy $\hat{\lambda}$ of the operator on the market for the case of a single aggregator ($J = 1$). In particular, we investigate how the total DR gain of our market, $\Delta c$, is allocated to the market participants. In Fig. 5(a) we observe that the operational cost of the operator is not monotonic in $\hat{\lambda}$, while electricity cost $c$ is decreasing in $\hat{\lambda}$. We depict also the optimal solution $\hat{\lambda}^{*}$ that we derive from the benchmark case. In contrast, in Fig. 5(b) we see that the DR gain $\Delta c$ is an increasing function of $\hat{\lambda}$ and the same holds for the total net profit of the aggregators and the net payoff of home users. The actual share of the arising DR benefits is depicted in Fig.5(c). Note that, for $\hat{\lambda} < 0.15$, the market derives zero DR gain. Hence, we depict the corresponding gains as zeros. In the particular setting, the users generally derive almost double the gains of the aggregators. In addition, at $\hat{\lambda}^{*} = 0.64$, all participants derive a significant portion of
the DR gains which is not true for extreme values of $\lambda$.

The exact amount of DR benefits that each entity receives, depends on the efficiency of the applied DR scheme and the inelasticity of user demands. In Fig. 6 we compare the performance of the full information approach and the proposed market mechanism as the average inelasticity of demands increases. In the following plots, the rightmost point of the x axis corresponds to the case of extreme inelasticity which coincides with the solution of no compensation (i.e. flat pricing). We observe that our approach sacrifices some of the operators’ gain for the sake of aggregators and users. Thus, although the operational cost of the operator is in general comparable (except for low inelasticity regime), our DR market provides a higher DR gain. This additional market gain is shared unevenly between the aggregator and the corresponding end-users. Compared to flat pricing though, the proposed DR market guarantees significant benefits for all the participating entities.

Next, we compare the performance of the proposed scheme of adaptive rewards $\lambda^k$ with that achieved when a nonprofit operator provides all the DR benefits to the lower levels through a constant reward $\hat{\lambda} = 1$. For a scenario of $J = 5$ aggregators, we depict in Fig. 7 and Fig. 8 the impact of inelasticity on the benefits of each entity. From Fig. 7(a) we observe that our DR scheme provides a reduction in the total operational cost of up to 15% in comparison to the case of flat pricing. However, the exact cost reduction depends strongly on the inherent inelasticity of the user demands. The same holds
for the power generation cost. Our scheme guarantees also that the aggregators and the users generally derive comparable financial benefits.

Interestingly, the net payoff of the users and the net profit of the aggregators are not monotonic in inelasticity parameter $v$. Instead the maximum is observed in the medium inelasticity regime. This can be easily justified. In the high inelasticity regime, the aggregators spend most of the reward provided by the operator in order to motivate end users to modify their demand pattern. This does not lead to increased net payoff for the users though, since the compensation provided is counteracted by the inconvenience caused. On the other end, low inelasticity leads to reduced negotiation power from the user side and consequently reduced income.

Similar results are derived from the scenario of a nonprofit operator, depicted in Fig. 7(b). However, here the total cost of the operator is constant, while small enough inelasticity enables the aggregators to provide low compensation to the end-users in order to acquire their DR services and hence exploit most of the DR benefits for themselves. By comparing the two approaches, we observe that although the benefits of the aggregators and the users increase in the case of a nonprofit operator, the electricity cost and the market gain remain almost the same. This indicates that our adaptive approach exploits most of the benefits arising from DR. However, as shown in Fig. 8 the portion of DR gain received by each entity differs significantly in the two approaches.

VI. RELATED WORK

In a series of works, the problem of demand response is mapped to that of selfish routing over parallel links, with each link corresponding to a timeslot. Indicatively, the authors of [7], [11] consider a set of users that have to schedule their demands within a finite horizon. Each user receives utility for each unit of energy consumed according to a concave utility function and is charged for this consumption by the operator. Thus, for given prices each user can calculate the consumption pattern that maximizes its net utility. Under the assumption that the operator’s objective is to maximize social welfare, i.e. the total utility minus the cost of supply, they derive convergent distributed algorithms based on auction mechanisms and dual decomposition methods respectively. In a similar framework, a two time scale wholesale electricity market is considered in [8], where power may be purchased either through a day ahead market or in real-time.

The works [7], [8], [11] assume a social-welfare maximizing operator that communicates and negotiates directly with utility-maximizing end-users. In this case, according to the fundamental theorem of welfare economics, the optimal strategy is to set prices equal to the marginal cost of supply. However, this does not hold for our scenario, where a self-interested operator seeks to minimize its operational cost. This has been considered lately in [6] that investigates the problem of demand scheduling from the operator’s perspective.
authors devise a stochastic model for demand requests which arrive according to a Poisson process, they have exponentially distributed power requirements, and they need to be activated within a deadline. They derive a threshold-based demand scheduling policy that is asymptotically optimal as the deadline expiration rate goes to zero. In contrast to our work, [6] assumes that the operator has direct control over the demands and hence they do not consider incentives for end-users. The DR scenario of a cost-minimizing operator that incentivizes home users to shift their demands through dynamic pricing has been considered in [12].

In addition, several works cast the problem of demand response as a Walrasian auction, where prices are set so as to match supply and demand, and use tatonnement mechanisms for its solution. Indicatively, the work [13] considers price-taking residential customers that schedule their consumption throughout the day and a utility operator that sets prices to maximize social welfare. A similar tatonnement process has been proposed in [14] for the stochastic version of this problem.

All the works mentioned above, require extensive message exchange between the operator and the end-users. Hence, scalability issues may arise in a large scale deployment. Hierarchical market structures, where aggregators serve as DR intermediates, appear as a promising approach to deal with scalability issues. To the best of our knowledge only a few works have considered the role of the aggregator in the future electricity market. Recently, the authors in [15] proposed that the aggregator should coordinate electricity generation and demand response. Given a reference supply / demand imbalance signal or an indirect signal in the form of time varying market price, the aggregator has to solve a convex quadratic program for each time slot. The authors of [4] propose a simplified hierarchical market mechanism, where the operator sets a target demand curtailment and the aggregator provides compensation to the end-users to meet this target at minimum payment. Each consumer is a price-taker and bids its supply function in order to minimize its disutility. In this setting, two bidding mechanisms are proposed that converge to the optimum. Both these works do not consider the incentives that would enable such a mechanism and consequently do not capture the interaction of competing aggregators.

To the best of our knowledge [5] is the only work that considers the interaction of several aggregators, but for the microgrid scenario. A two stage market model is proposed, where in the first stage the price paid by the operator and the electricity provided by each aggregator is decided through a tatonnement process. For the lower stage a supply function bidding mechanism is proposed, where the microgrids bid their supply functions and the aggregator sets the price so as to maximize its profit. While [5] focuses on microgrid generation within a particular time slot, here we focused on the scheduling of residential demands across time.

The importance of DR in the residential sector was also quantified recently in [16], where it was shown that a slight extension of 10% in the total operation time of residential demands may reduce peak consumption by 125MW. Here, we considered in a similar framework the discomfort caused to the users and the incentives required to achieve such DR benefits. In order to quantify the benefits of a residential DR scheme, a realistic model of the home energy consumption is required. For this purpose, the authors of [10] developed a generator of per appliance consumption daily patterns. In this work, we extend this generator by incorporating demand elasticity parameters.

VII. CONCLUSIONS

The main contribution of our work is that it provides a better understanding of the role of the aggregators in the future smart grid. We focused on a hierarchical market structure and investigated the interaction of end-users, the utility operator, and several aggregator entities that act as intermediaries. Under the assumption of price-taking and self-interested users, we characterized the optimal solution from the operator point of view that seeks to minimize its operational cost. In this benchmark scenario the operator has access to various parameters pertaining to the demand of end-users. We also proposed a market model that leads to significant benefits for all the involved entities.
Based on realistic demand traces we demonstrated that access to user information enables the operator to exploit most of the DR benefits for itself, while the proposed non-cooperative market mechanism leads to a more fair allocation of the DR benefits. Interestingly, our numerical evaluation indicates that in a hierarchical market, the utility of the end-users in not a monotonously increasing function of elasticity. Here, we assumed that the users truthfully respond to the announced compensation from the aggregators. However, since additional benefits may arise through strategic misreporting, the investigation of such user strategies and the derivation of mechanisms that guarantee truthfulness are interesting topics for future study.

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