Optimal Energy Storage Control Policies for the Smart Power Grid

Iordanis Koutsopoulos[†*] Vassiliki Hatzi[†] Leandros Tassiulas[†*]
[†]Department of Computer and Communications Engineering, University of Thessaly
[*]Center for Research and Technology Hellas (CERTH), Greece

***Abstract*—Electric energy storage devices are prime candidates for demand load management in the smart power grid. In this work, we address the optimal energy storage control problem from the side of the utility operator. The operator controller receives power demand requests with different power requirements and durations that are activated immediately. The controller has access to one energy storage device of finite capacity. The objective is to devise an energy storage control policy that minimizes long-term average grid operational cost. The cost is a convex function of instantaneous power demand that is satisfied from the grid, and it reflects the fact that each additional unit of power needed to serve demands is more expensive as the demand load increases.**

For the *online dynamic control* problem, we derive a threshold-based control policy that attempts to maintain balanced power consumption from the grid at all times, in the presence of continual generation and completion of demands. The policy adaptively performs charging or discharging of the storage device. The former increases power consumption from the grid and the latter satisfies part of the grid demand from the stored energy. We prove that the policy is asymptotically optimal as the storage capacity becomes large, and we numerically show that it performs very well even for finite capacity. The *off-line problem* over a finite time horizon that assumes a priori known power consumption to be satisfied at all times, is formulated and solved with Dynamic Programming. Finally, we show that the model, approach and structure of the optimal policy can be extended to also account for a renewable source that feeds the storage device.

I. Introduction

The smart power grid will rely on information and communication technologies and advanced control methods to manage the dynamic demand load and to ensure efficient use of electric energy [1], [2]. Major constituent entities of the smart grid are expected to be various types of renewable energy sources, as well as energy storage components that may or may not be attached to the renewable ones. Smart metering and bidirectional communication enable real-time interconnection of the consumer and operator premises through IP addressable components over the Internet. These technologies allow power consumption monitoring, automated control of consumption of customer appliances through messages from the operator Command and Control (C&C) center, real-time electricity price signaling, and fault diagnosis.

Demand load management is primarily employed by power utility operators so as to reduce the grid operational costs. The rationale of demand load control is to alleviate high demand load at peak times. This can be achieved for instance by using the time slack of delay-tolerant power demands so as to temporally shift part of the peak load in time when it is feasible to do so. Thus, the risk of a potential grid failure is reduced, while the operational cost is lowered by avoiding using more expensive or less efficient power generation means.

Recent advances in electric energy storage technologies have rendered backup devices like uninterrupted power supply (UPS) or batteries, and Plug-in Hybrid Electric Vehicles (PHEVs) prime candidates for demand load management. These devices have significant storage capacity of up to a few tens of KWh. With appropriate storage management policies, these devices can be quite advantageous for electric utility operators and consumers. If stored energy management is delegated to the *grid operator*, a valid objective is to minimize the grid operational cost. Batteries can be charged at off-peak-load times, and (part of) this stored energy can be used to satisfy increased demand at peak times. If storage management is performed at the *consumer* level (e.g. through PHEVs), the goal is to minimize the cost of power consumption, assuming that an instantaneous time-of-use price per unit of consumed power is fed back. Energy can then be stored when the price of consumed power is low, and it can be used to satisfy part of the demand when the price is high.

In this paper, we address the problem of *optimal stored energy control* faced by the utility operator whose objective is to *find an energy storage management policy for the storage device such that the grid operational cost over a time horizon is minimized*. The operational cost is modeled as a convex function of instantaneous total power consumption so as to reflect the fact that each additional Watt of power needed to serve power demands becomes more expensive as the total power demand increases.

A. Related work

There exists significant amount of work on leveraging stored energy in various contexts. In wireless networks, the work [3] considers optimal control with rechargeable batteries and time-varying channels, in the sense of maximizing total utility which is a concave function of link rates. Energy is obtained from the battery, and the transmit power is controlled for transmitting over wireless links of time-varying state. The authors propose a policy that is asymptotically optimal for sufficiently large battery capacity. A similar problem is considered in [4], along with a detailed assessment of the quality of a solution versus the energy buffer size, and a policy is proposed that performs well even for smaller buffer sizes. Two online algorithms are developed, which jointly manage the energy and make power allocation decisions for transmissions.

In the context of electric energy storage, the work [5] solves the optimal stored energy control from the consumer side amidst time-varying prices. The work [6] uses UPS storage devices to reduce the electricity bill in a data center under varying prices. An online algorithm is proposed, based on Lyapunov optimization, for optimizing the time average cost which is linear in power demand load. Various constraints on charging and discharging, as well as the cost of repeated charging and discharging are modeled. In [7], the interaction of multiple energy storage facilities is considered. The strategy of each facility is how much to charge or discharge, and the different entities interact through a formed price that is linear in total grid load. In [8], the authors consider coordinated charging of PHEVs so as to use stored electric energy to cope with high demand and intermittent generation capability of renewable sources. Finally, in [9], the authors consider the problem of minimizing the time average cost of using resources other than the basic ones, subject to keeping the queue of demand backlog stable, in the presence of renewable supply sources. The cost is again linear in the amount of energy consumed. Various research initiatives that rely on efficient energy storage exist, such as the WINSmartGrid project [10] at UCLA and Vehicle to Grid (V2G) [11] at University of Delaware.

B. Our contribution

We address the optimal energy storage control problem that is faced by a grid operator. To the best of our knowledge, this is the first work that considers the problem from the point of view of the operator. The controller has access to one energy storage device of finite storage capacity. The contributions of this work to the literature are as follows: (i) We model operational cost as a convex function of instantaneous power demand that is satisfied from the grid, which captures the increasing marginal cost for the operator as demand load increases; (ii) we study the online dynamic storage control problem by devising a stochastic model for continually generated demands and completions, and we consider minimizing long-term average cost. This model is also novel in the literature. We derive a threshold control policy that attempts to maintain balanced power consumption from the grid by adaptively managing the storage device charge and discharge processes, and by satisfying part of the demand from the grid or the storage device. We prove that the policy is asymptotically optimal as the battery storage capacity becomes large, and we numerically show that it performs quite well even for finite storage capacity; (iii) We study the off-line control problem over a finite time horizon, where the power consumption to be satisfied at each time is known a priori, and we use Dynamic Programming to solve it; (iv) we extend the model, approach and structure of the optimal policy to account for a renewable source that feeds the storage device.

The rest of the paper is organized as follows. In section II we study the online control problem; we show the optimality of the proposed threshold policy, and we present an extension that includes a renewable source which feeds the storage device. In section III we solve the off-line problem, in section IV we present numerical results, and in section V we conclude our study. The terms “battery” and “storage device” are used interchangeably with the same meaning in the paper.

II. The Online Energy Storage Control Problem

A. System model

1) Power demand arrival and service processes: In the online version of the problem, power demand requests are generated continually and arrive at the grid operator controller according to a Poisson process, with average rate λ requests per unit of time. The time duration s_n of each power demand request $n = 1, 2, \ldots$ is a random variable that is exponentially distributed with parameter s, i.e. $\Pr(s_n \le x) = 1 - e^{-sx}$, $x \ge 0$. Equivalently, the mean request duration is $1/s$ time units, and s is the average service rate for power demand tasks. Further, let p_n denote the power requirement of demand n. These assumptions are motivated for mathematical tractability as they facilitate the derivation of the optimal policy, yet they are also close to reality, as they capture the burst of arriving requests and their different durations. Denote by $P(t)$ the amount of power consumption on the grid at time t as a result of the process above.

Consider the system above, first without the storage device. If the power requirement of each task is 1, the instantaneous power consumption, $P(t)$ is the number of active demands at t. Under the assumptions stated above for the arrival and service processes, $P(t)$ is a continuous-time Markov chain. In fact, since each power demand task is activated upon arrival, $P(t)$ is *the occupation process of an $M/M/\infty$ service system*. From state $P(t)$, there are transitions to state:

- $P(t) + 1$ with rate λ, when new demand requests arrive.
- $P(t) - 1$ with rate $P(t)s$, when one of the current $P(t)$ active demands is completed.

The steady-state probabilities $q_i = \lim_{t\to\infty} \Pr(P(t) = i)$, for $i = 1, 2, \ldots$, on the number of active power demand tasks are obtained from equilibrium equations as:

$$q_i = \left(\frac{\lambda}{s}\right)^i \cdot \frac{e^{-\lambda/s}}{i\,!}\,, \tag{1}$$

which is Poisson distributed with parameter $\frac{\lambda}{s}$. Thus, the expected number of active requests at steady state is $\mathbb{E}[P(t)] = \frac{\lambda}{s}$, where the expectation is with respect to the stationary Poisson distribution of $P(t)$.

The extension to different power requirements of demands goes as follows. The power requirement of each demand task, $\hat{P}$ is a random variable with a discrete or continuous probability distribution, and it is independent from process $N(t)$ that shows the number of active demands. Let $\mathbb{E}[\hat{P}]$ be the average task power requirement. Power consumption at time t is $P(t) = \hat{P} \cdot N(t)$, and the average power consumption at steady state is $\mathbb{E}[P(t)] = \frac{\lambda}{s}\mathbb{E}[\hat{P}]$.

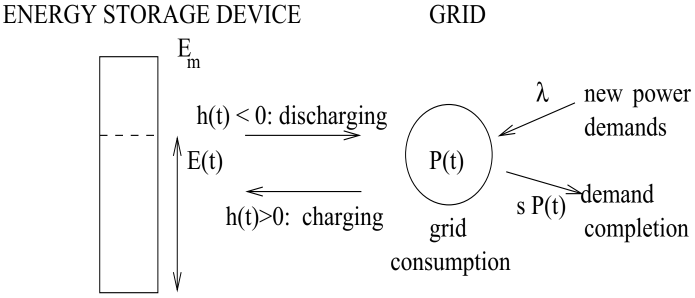

Fig. 1. Overview of system model with the energy storage device, the charging and discharging process, and the interaction with grid consumption.

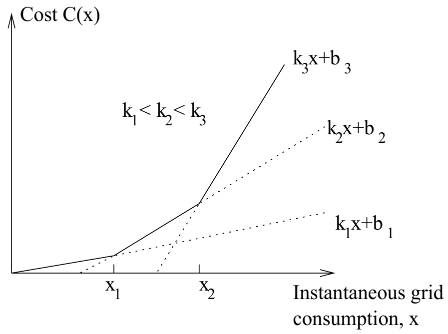

Fig. 2. The cost as a piece-wise linear convex function of the grid load.

2) Energy Storage Device: There exists a electric storage device (battery) of storage capacity E_m KWhs available to the grid operator. Time is divided into slots of unit size. At each time slot t, the battery may be charging or discharging. Let $E(t)$ be the stored amount of energy at the beginning of slot t. Define the decision variable $h(t)$ as the *rate* at which the battery is charged at time t, with the following convention. If $h(t) > 0$, the battery charges, and energy flows into it from the grid with rate $h(t)$. This in turn implies that the total grid consumption is $h(t)$ plus the power demand $P(t)$ that is served due to requests arising as above. Thus the total amount of power demand load that is served by the grid is $P(t)+h(t)$. If $h(t) < 0$, the storage device discharges, namely energy flows out of the battery at rate $|h(t)|$, and it is used to serve part of the demand load $P(t)$ on the grid. Hence, the amount of power demand that is actually served from the grid is $P(t) + h(t) < P(t)$, since a portion of the power demand is served from the battery. The model is depicted in Fig. 1.

The level of stored energy at the battery evolves with time as $E(t+1) = E(t) + h(t)$. Since it is $0 \le E(t) \le E_m$ at all times t, it becomes evident that $h(t)$ must satisfy:

$$-E(t) \le h(t) \le E_m - E(t)\,, \forall\, t\,. \tag{2}$$

In order to demonstrate our approach, we assume there exist no further constraints on maximum charging and discharging rate other than those implied by (2). We also assume there exists no switching cost in terms of delay for transferring power demands from the grid to the battery and vice versa. The former transfers take place when we decide to discharge the battery, and the latter occur when the battery empties while some tasks are served.

3) Cost Model: Let $X(t) = P(t) + h(t)$ denote the total consumed power on the grid at time t. This is the sum of active demand tasks that are served by the grid, plus the charging rate of the battery, if the latter is positive. We denote the instantaneous *operator cost* associated with power consumption $X(t)$ at time t as $C(X(t))$, where $C(\cdot)$ is an increasing, differentiable convex function. Convexity of $C(\cdot)$ reflects the fact that the *differential* cost of grid power consumption for the electric utility operator increases as demand increases. That is, each unit of additional power needed to satisfy the increasing demand becomes more expensive to obtain and make available to the consumer. For instance, supplementary power for serving high demand may be generated from expensive sources (e.g. gas micro-turbines), or it may be imported at high prices from other countries. The cost may also be a piecewise linear function, where the different slopes of the linear segments represent different classes of power consumption (Fig. 2).

B. *Problem formulation*

At the beginning of each time slot t the controller observes the total grid consumption level $X(t)$ (and thus, also $P(t)$), and the energy level $E(t)$. The communication between the controller and the battery takes place through a high-speed connection with zero delay via a smart device attached to the battery which keeps track of battery level $E(t)$. At each time slot t, the controller needs to decide whether it will charge or discharge the battery and how much. If the battery will be discharged, part of the demand $P(t)$ on the grid is served from the battery energy, and $X(t) < P(t)$ If the battery will be charged, the load on the grid is $P(t)$ plus the decided charging rate of the battery.

Denote the system state at time t as $\mathbf{x}(t) = (P(t), E(t))$. For now, assume that all quantities are restricted to integer values. Let the initial state be $\mathbf{x}(0) = (P(0), E(0))$. The long-run average cost associated with a policy π is:

$$J_\pi(\mathbf{x}(0)) = \lim_{T\to+\infty} \mathbb{E}^{\pi}_{\mathbf{x}(0)}[\frac{1}{T}\sum_{t=0}^{T-1} C\big(P(t)+h(t)\big)]\,, \tag{3}$$

where the expectation is with respect to the randomness of $P(t)$ and the one induced by a policy π on the state process that starts from state $\mathbf{x}(0)$. A policy π is a way of selecting variables $\{h(t)\}_{t=0,1,\ldots}$ subject to the evolution equation, $E(t+1) = E(t) + h(t)$, and subject to constraint (2).

A policy π^* is optimal if it minimizes (3) over all policies satisfying the constraints above. Under the assumptions above on arrival and completion process, the problem of minimizing the long-run average cost (3) is cast as a discrete-time Markov Decision Process (MDP).

Equivalently to the problem of minimizing the cost in (3), we may consider the corresponding problem associated with the β-discounted cost, where $0 < \beta < 1$ is a discount factor.

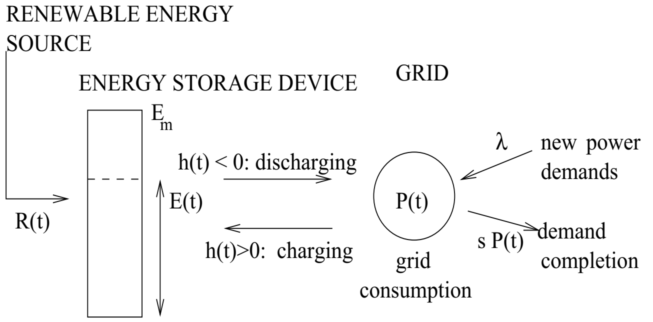

Fig. 3. Extension to the model, with a renewable source feeding the battery.

Theβ-discounted cost for policy π is defined as:

$$V_\pi^\beta(\mathbf{x}(0)) = \mathbb{E}_{\mathbf{x}(0)}^\pi [\sum_{t=0}^{\infty} \beta^t C\big(P(t)+h(t)\big)] \,. \tag{4}$$

The dynamic programming equation is:

$$\begin{aligned} V(P(t),E(t)) = & \min_{-E(t)\leq h(t)\leq E_m-E(t)} \big\{ C\big(P(t)+h(t)\big) \\ & +\beta\lambda C\big(P(t)+1, E(t)+h(t)\big) \\ & + \beta s P(t) C\big(P(t)-1, E(t)+h(t)\big) \big\} \,. \end{aligned} \tag{5}$$

C. An Asymptotically Optimal Control Policy

Instead of proceeding with solving the MDP problem above, we derive a simple policy and show that it is asymptotically optimal for large storage capacity values. In section V we numerically verify that the policy performs quite well even for finite storage capacity values.

Consider the following *dynamic control* policy. There exists a threshold, P_0. Each time a new demand request arrives, the controller checks $P(t)$. If $P(t) \leq P_0$, then all active demand requests are served by the grid energy, and a decision to *charge* the battery is taken, with charging rate $\tilde{h}(t) = P_0 - P(t)$. If $P(t) = P_0$, then the battery is not charged. If the charging decision is taken say at time t_1, and charging takes place between times t_1 and t_2 (where t_2 is dictated by a new event occurrence such as arrival or service completion, or empty battery), the battery energy level is $E(t_2) = E(t_1) + \tilde{h}(t_1)(t_2 - t_1)$.

On the other hand, if $P(t) > P_0$, a decision to *discharge* the battery is taken, with rate $\tilde{h}(t) = P(t) - P_0$, until the next event. Again, if discharging takes place between times t_1 and t_2 and the initial decision to discharge was at t_1, then $E(t_2) = E(t_1) - \tilde{h}(t_1)(t_2 - t_1)$. Each time a demand request that is served by the grid is completed, or the battery energy becomes zero, the controller again checks $P(t)$ and the same control about battery charging or discharging is applied. Whenever $E(t) = 0$ during the procedure of discharging, the controller returns back to the grid the requests that were served by the battery at that time t. The policy described above is summarized as follows:

- If $P(t) \leq P_0$, charge the battery with rate $P_0 - P(t)$.
- If $P(t) > P_0$, discharge the battery with rate $P(t) - P_0$.

Since $P_0 - P(t)$ can take positive or negative values, the policy above, which we denote by (P) is succinctly described as:

$$h(t) = \max\big\{ - E(t), P_0 - P(t) \big\} \,. \tag{6}$$

As will be shown next, the optimal threshold is $P_0 = P_{av} = \mathbb{E}[P(t)] = \frac{\lambda}{s}$. Note that for the long-term average cost, we have the following lower bound from Jensen's inequality:

$$\mathbb{E}[C\big(P(t)+h(t)\big)] \geq C\big(\mathbb{E}[P(t)+h(t)]\big) \,. \tag{7}$$

Theorem 1: Policy (P) is asymptotically optimal, in the sense that its performance converges to the lower bound (7) as $E_m \to \infty$, and therefore it minimizes the long-term average cost (3).

Proof: We provide a sketch of the proof. The sequence of events defined by $\{P(t) - P_0 \geq E(t)\}$ has diminishing probability as $E_m \to \infty$. This is because the values of $E(t)$ increase in general as well. Recall that the feasible controls set at each time t is such that $-E(t) \leq h(t) \leq E_m - E(t)$. For increasing values of energy storage E_m, $h(t)$ takes values in the *interior* of this set with probability that approaches 1. Indeed, $h(t) < E_m - E(t)$ as $E_m \to \infty$, since the battery capacity increases. Furthermore, due to the previous argument, it is $h(t) > -E(t)$ with probability that approaches 1. This means that, while discharging the battery, there is always sufficient energy such that the battery is almost never empty.

For the control process $h(t)$, we verify that $\lim_{E_m\to\infty} \mathbb{E}[h(t)] = 0$, since the battery charges with rate $P_0 - P(t)$ whenever $P(t) < P_0$, and it discharges with rate $P(t) - P_0$ when $P(t) > P_0$. For $P_0 = P_{av} = \mathbb{E}[P(t)] = \frac{\lambda}{s}$, the charging and discharging events each take place for half the amount of time in the long-run on average, and thus $\mathbb{E}[h(t)] \to 0$. Therefore, under that policy, the consumption from the grid is constant and equal to $\mathbb{E}[P(t) + h(t)] = \frac{\lambda}{s}$, and the lower bound (7) is reached. ∎

D. Extension to the model: Renewable source

We consider the following extension to the model. There exists a renewable energy source which feeds the battery. Such a setup is expected to be commonplace in the future, with photovoltaic or other renewable sources attached to the battery. Denote by $R(t)$ the renewable source energy generation process, which is assumed to be an arbitrary stationary process with expectation $R_{av} = \mathbb{E}[R(t)]$. The rest of the model is the same as the one above, and the system is depicted is Fig. 3. The battery energy level equation is:

$$E(t+1) = E(t) + R(t) + h(t) \,, \tag{8}$$

and the feasible controls set at time t is such that, $-R(t) - E(t) \leq h(t) \leq E_m - E(t) - R(t)$. This system is equivalent to one without the renewable source, where $h(t)$ is substituted by $h(t) - R(t)$. It can be shown that the policy that minimizes long-term average cost for the setup with a renewable source is as follows:

- If $P(t) \leq P_{av} - R_{av}$, charge the battery with rate $P_{av} - R_{av} - P(t)$.
- If $P(t) > P_{av} - R_{av}$, discharge the battery with rate $P(t) - P_{av} + R_{av}$.

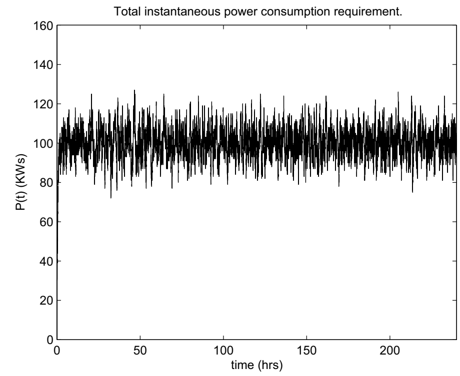

Fig. 4. Total instantaneous power demand, $P(t)$.

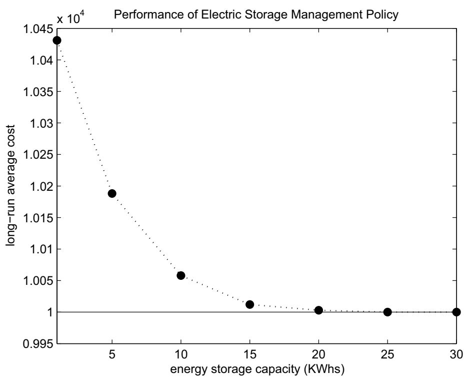

Fig. 5. Performance of the proposed energy storage control policy as a function of available storage capacity.

Observe that if $P_{av} - R_{av} < 0$, battery charge will not take place since in that case the renewable source fully covers the power demand requirements.

III. The Off-line Problem

In this section, we will use notation P_t, E_t, X_t, h_t instead of $P(t), E(t), X(t), h(t)$. In the off-line problem, we are given the initial battery energy E_0 and power consumption levels $P_0, \dots, P_{T-1}$, and we want to find the sequence of controls $h_0, \dots, h_{T-1}$ to minimize the total cost,

$$\sum_{t=0}^{T-1} C(P_t + h_t)\,, \tag{9}$$

subject to $E_{t+1} = E_t + h_t$ and $-E_t \le h_t \le E_m - E_t$. The Bellman equation for the cost-to-go at stage $T-1$ writes

$$J_{T-1}(P_{T-1}, E_{T-1}) = \min_{-E_{T-1} \le h_{T-1} \le E_m - E_{T-1}} C(P_{T-1} + h_{T-1}). \tag{10}$$

Since $C(\cdot)$ is increasing, the minimizing value of h_{T-1} is $h^*_{T-1} = -E_{N-1}$, and thus $J_{T-1}(P_{T-1}, E_{T-1}) = C(P_{T-1} - E_{T-1})$. For $t = 0, \dots, T-2$, the Bellman equation for the cost-to-go is:

$$J_t(P_t, E_t) = \min_{-E_t \le h_t \le E_m - E_t} [C(P_t + h_t) + J_{t+1}(P_{t+1}, E_t + h_t)] \tag{11}$$

For each t, the minimizing h_t^* is:

- $h_t^* = -E_t$ (discharge until the battery is empty), or
- $h_t^* = E_m - E_t$ (charge until it is full), or
- $h_t^* \in (-E_t, E_m - E_t)$, in which case the derivative of the expression in (11) in the brackets with respect to h_t is set to zero.

The optimal control at that stage will emerge by comparing the cost to go for the three cases above and choosing the solution that corresponds to the minimum cost-to-go.

For $t = T-2$, by applying (11) together with $E_{T-2} + h_{T-2} = E_{T-1}$, we get:

$$\begin{aligned}
J_{T-2}(P_{T-2}, E_{T-2}) &= \min_{-E_{T-2} \le h_{T-2} \le E_m - E_{T-2}} [C(P_{T-2} + h_{T-2}) \\
&\quad + J_{T-1}(P_{T-1}, E_{T-2} + h_{T-2})] \\
&= \min_{-E_{T-2} \le h_{T-2} \le E_m - E_{T-2}} [C(P_{T-2} + h_{T-2}) + C(P_{T-1} - E_{T-1})] \\
&= \min_{-E_{T-2} \le h_{T-2} \le E_m - E_{T-2}} [C(P_{T-2} + h_{T-2}) \\
&\quad + C(P_{T-1} - E_{T-2} - h_{T-2})]\,.
\end{aligned}$$

There are three possibilities:

1) The minimizing $h^*_{T-2} = -E_{T-2}$. Then the cost $J(P_{T-2}, E_{T-2})$ is $C(P_{T-2} - E_{T-2}) + C(P_{T-1})$.
2) The minimizing $h^*_{T-2} = E_m - E_{T-2}$. Then the cost is $C(P_{T-2} + E_m - E_{T-2}) + C(P_{T-1} - E_m)$.
3) The minimizing $h^*_{T-2} \in (-E_{T-2}, E_m - E_{T-2})$. In that case, by setting the derivative of the cost to zero, we find:
$$h^*_{T-2} = \frac{P_{T-1} - P_{T-2} - E_{T-2}}{2} \tag{12}$$
and the cost is $2C(\frac{P_{T-1} + P_{T-2} - E_{T-2}}{2})$.

We can proceed backwards in the same fashion. At each stage t, we substitute $J_{t+1}(\cdot)$. When reaching stage 0, we can compute h_0^* since we know E_0. We can then go forward by replacing that to find $h_1^*, \dots, h^*_{T-1}$. Similarly to the online problem, the solution balances as the derivative of cost as much as possible, given the battery capacity constraints.

IV. Numerical Results

In order to evaluate the performance of our policy for finite storage capacity, we first compute the long-term average cost $E[C\big(P(t) + h(t)\big)]$ and compare it to the lower bound in (7). Our simulation scenario ran for a horizon $T = 240$hrs. The power demand arrival and service processes are Poisson with average arrival rate $\lambda = 200$ requests / hour and average service rate $s = 2$ requests / hour, i.e. the average demand duration is $1/s = 1/2$ hour. Also, the average power requirement per demand was $1KW$. The power demand over time is depicted in Fig. 4. Thus, the average demand load is $E[P(t)] = \frac{\lambda}{s} = 100$KW. We consider cost function $C(x) = x^2$.

In Fig. 5, we show the cost of the proposed policy (P) for different values of storage capacity E_m. Recall that policy (P) was constructed by ignoring storage capacity constraints, and it seeks to maintain balanced power consumption at all times. It can be observed that the performance of the policy is optimal for battery capacity $E_m \ge 24$KWhs, since the resulting

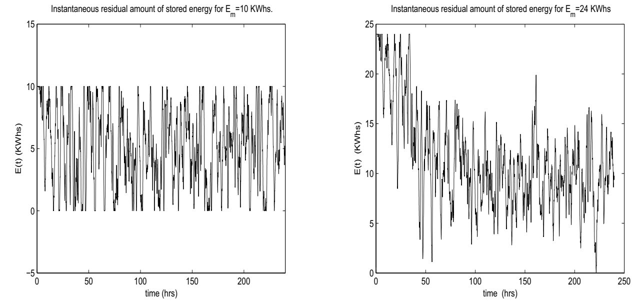

Fig. 6. Instantaneous residual amount of stored energy for $E_m = 10$KWhs and $E_m = 24$KWhs.

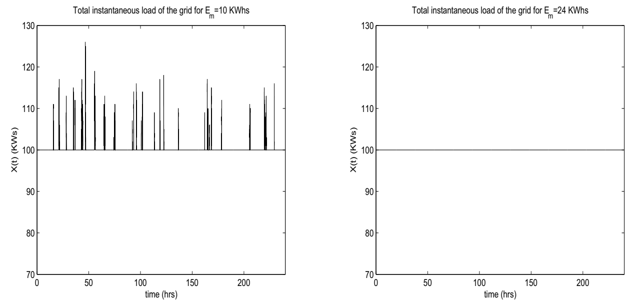

Fig. 7. Total instantaneous grid load for $E_m = 10$KWhs and $E_m = 24$KWhs.

cost for these values reaches the lower bound. As anticipated, the average cost decreases as E_m increases, and it ultimately converges to the lower bound, $(\lambda/s)^2$.

Next, we compare two scenarios. One with battery capacity $E_m = 10$KWhs and one with $E_m = 24$KWhs. In Fig. 6 we show instantaneous residual stored energy for the cases above. When $E_m = 10$KWhs, the battery is fully discharged (empty) for a total of 69 times. On the contrary, for $E_m = 24$KWhs, the battery is never fully discharged. In Fig. 7 we depict the total instantaneous load on the grid, $X(t) = P(t) + h(t)$ for the two cases above. Observe that if $E_m = 10$KWhs, the total load exceeds $P_{av} = 100$ for a total of 69 times as well. Clearly, these are the times when the battery empties, and the load that the battery was serving is moved to the grid. On the contrary, for $E_m = 24$KWhs, the battery is never fully discharged, and $X(t)$ remains constant and equal to $P_{av} = 100$. Charging and discharging for both cases are shown in Fig. 8.

We also seek the value of the minimum required storage capacity for which our policy reaches the lower bound in (7), as a function of demand load λ/s. Fig. 9 presents our findings. The minimum required battery capacity can be seen to increase in a concave-like fashion in four out of the five points.

V. Conclusion

We proposed a storage control policy that is asymptotically optimal for large storage capacity and performs quite well even for finite capacity values. In the future, we plan to compute explicitly the optimal policy for finite capacity. An enhanced model that would include e.g. a switching cost due to frequent battery charging and discharging is also worth studying.

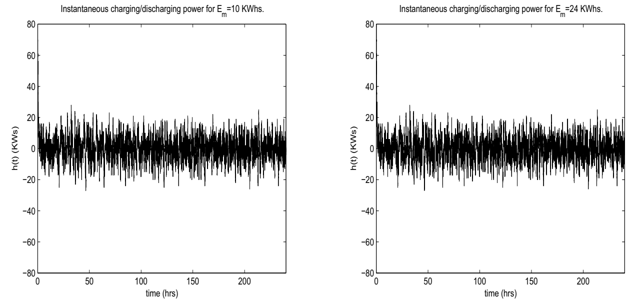

Fig. 8. Instantaneous charging / discharging power for $E_m = 10$KWhs and $E_m = 24$KWhs.

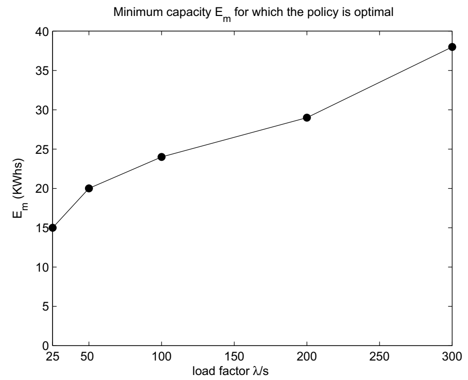

Fig. 9. Minimum required capacity E_m for which the policy is optimal, versus the demand load.

VI. Acknowledgment

The authors acknowledge the support of the European Commission through the NoE project TREND (FP7-257740).

References

[1] S. Keshav and C. Rosenberg, "How Internet Concepts and Technologies can help Green and Smarten the Electrical Grid", *Proc. ACM SIGCOMM Green Networking Workshop,* 2010.

[2] M. G. Morgan, J. Apt, L. B. Lave, M. D. Ilic, M. Sirbu, and J. M. Peha, "The many meanings of smart grid", *Tech. Rep., Carniege Mellon University,* 2009.

[3] M. Gatzianas, L. Georgiadis, and L. Tassiulas, "Control of wireless networks with rechargeable batteries", *IEEE Trans. Wireless Comm.,* vol. 9, no. 2, pp. 581-593, Feb. 2010.

[4] L. Huang and M.J. Neely, "Utility optimal scheduling in energy harvesting networks", *in Proc. Mobihoc,* 2011.

[5] P. van de Ven, N. Hegde, L. Massoulié and T. Salonidis, "Optimal control of residential energy storage under price fluctuations", *Proc. IARIA Energy Conf.,* 2011.

[6] R. Urgaonkar, B. Urgaonkar, M.J. Neely and A. Sivasubramaniam, "Optimal power cost management using stored energy in data centers", *in Proc. SIGMETRICS,* 2011.

[7] P. Vytelingum, T.D. Voyce, S.D. Ramchurn, A. Rogers and N.R. Jennings, "Agent-based micro-storage management for the smart grid", *Proc. 9th Int. Conf. on Aut. Agents and Multiagent Sys. (AAMAS)*, 2010.

[8] M. C. Caramanis and J. Foster, "Management of Electric Vehicle Charging to Mitigate Renewable Generation Intermittency and Distribution Network Congestion", *48th IEEE Conf. Dec. Control (CDC),* 2009.

[9] M.J. Neely, A. Saber Tehrani and A. G. Dimakis, "Efficient Algorithms for Renewable Energy Allocation to Delay Tolerant Consumers", *IEEE Int. Conf. on Smart Grid Commun. (SmartGridComm),* 2010.

[10] http://www.winmec.ucla.edu/SmartGrid/

[11] http://www.udel.edu/V2G/