Operator Collusion and Market Regulation Policies for Wireless Spectrum Management

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Abstract—The liberalization of wireless communication services markets and the subsequent competition among network operators, is expected to foster optimal utilization of the scarce wireless spectrum and ensure the provision of cost-efficient services to users. However, such markets may function inefficiently due to collusion of operators which yields a de-facto monopoly. Although it is illegal and detrimental to the users, creation of such cartels arises often in the form of implicit price fixing. In this paper, we consider a general such market where a set of operators sell communication services to a large population of users. We use an evolutionary game model to capture the user dynamics in selecting operators, under limited information about the actual service quality, and we analyze the anticipated interaction of the operators using coalitional game theory. We define a coalition formation game in order to rigorously study the conditions that render monopolistic or oligopolistic markets stable under different notions of coalition stability. We also provide direct and indirect regulation methods, such as setting price upper bounds or allocating different amounts of spectrum, in order to discourage undesirable equilibriums. Our approach provides intuitions about collusion strategies, as well as on directions for identifying and preventing them.

Index Terms—Operator collusion, evolutionary game theory, operator selection, coalitional game theory, spectrum management

1 Introduction

1.1 Motivation

The last few years we are witnessing an impressive transformation of the market landscape for wireless communication services. Namely, the ever increasing demand for ubiquitous network access and various IP-based services has led to an unforseen mobile data growth [1], [2]. This unprecedented network traffic is challenging traditional mobile network operators (MNOs) which are often forced to upgrade their technology, invest in spectrum, and deploy denser networks. At the same time, a number of mobile virtual network operators (MVNOs) [3], [4], have recently appeared offering cost-efficient communication services. Some notable examples are Republic Wireless [5] and Karma [6], which complement cellular access with Wi-Fi networks so as to reduce operating expenditures and attract new users, respectively.

Moreover, today there are solely Wi-Fi based communication solutions, such as the Wi-Fi network sharing communities (e.g., FON [7]), that offer alternative and low-cost network access to mobile or nomadic users, with increasing coverage in urban areas. This is an additional Internet access option for mobile users, especially those not having stringent quality requirements. This abundance of options is particularly important nowadays where the selection of service providers[1] is easier, often without involving long-term binding contracts, and can be realized in a dynamic fashion depending on the preferences of users.

In this new era, it was expected that competition among the different MNOs, in light of the available alternative communication solutions, would yield high quality services (QoS) offered at low prices, thus benefiting the users and facilitating related business activities (e.g., deployment of new content delivery services). Although to a large extent this expectation has been realized, notable cases of malfunctioning markets have been observed due to collusion of operators. These phenomena, despite being illegal, are often observed in real life in the form of price fixing which yields an effective monopoly, e.g., see [8], [9], and [10], and eventually higher charged prices and/or low-quality (e.g., in terms of bandwidth) services for the users [11].

Clearly, this multifaceted problem can potentially hamper the efficient operation of the communication markets and needs to be carefully studied. From a regulatory or policy-making perspective, it is important to be able to detect whether there is adequate competition in the market, mainly among the large MNOs. The key question is if the bundles of service quality and charged prices offered to the users are the result of price fixing, and how such collusion strategies are affected by the existence of alternative communication options. Equally important is to devise efficient methods so as to deter such phenomena. These methods can be direct, e.g., by setting upper price bounds, as happens in other malfunctioning markets, or indirect, e.g.,

1. Henceforth, we will often use the term *service provider* to refer to all these communication solutions.

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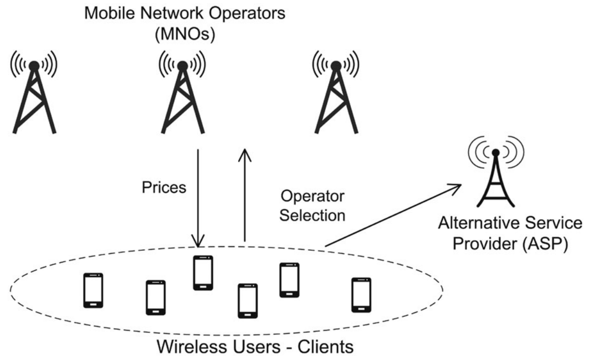


Fig. 1. Operators provide communication services of certain quality at different prices to a common pool of users. The ASP models the alternative choice that users have to satisfy their communication needs, e.g., companies as Republic Wireless [5] and FON [7].

through regulating the spectrum—and eventually the capacity—of the cellular networks.

Somehow surprisingly, despite their importance [12], [13], these phenomena and questions have attracted only limited attention from the communication networks' research community. Therefore, previous studies (mainly by economists) did not focus on the technical aspects of these markets (e.g., such as the co-existence of different type of networks). Our goal in this work is to take a more fine-grained modeling and analysis approach, and provide a systematic methodology for *(i)* modeling these emerging wireless communication services markets that encompass diverse solutions for the end users, *(ii)* analysing collusion phenomena among the key market players (i.e., the MNOs) while taking into account the end-user network selection strategy, and *(iii)* proposing respective regulatory approaches tailored for this type of wireless communication markets.[2]

Methodology and contributions. In particular, we consider a market with a set of typical MNOs and an alternative service provider (ASP) who represents the non-conventional communication solutions (e.g., a Wi-Fi network or an MVNO), as shown in Fig. 1. We focus on the MNOs who are competing to attract subscribers from a large pool of users. Each MNO has a given service capacity, which determines the offered QoS, and decides the charged price with the goal to increase its revenue. The users are free to choose one of the MNOs, or abstain from the main market and select an alternative solution. Their decisions are made based on their net utility, which is defined as the quality of service they receive minus the charged price. If all the MNOs are offering low utility, then the users opt for the ASP which ensures a minimum (or, *reservation*) utility.

Such a market inherently operates in two different time scales: a small one where users update their network selection, and a larger one where the MNOs determine their pricing policy, anticipating both the users' decisions and the strategy of the other MNOs. Given the complexity of this setting, we model the users' interactions through an evolutionary game theoretic framework, where, instead of modeling individual users, we analyze the dynamics of user populations distributed over the operators. These dynamics are a result of users' myopic (or, suboptimal) decisions since they only have limited information about the networks' capacity.

The MNOs may collude with each other and set equal prices, if this is expected to increase their revenues. Different coalitions compete to attract more users, and the users within each coalition are distributed to the MNOs, based on the capacity of each network (so as to ensure equal user utilities). Clearly, the actions of each coalition affect the revenue of MNOs in other coalitions as well. We model these interactions as a coalition formation game and study the properties and stability of the resulting collusion equilibriums. Moreover, we consider different notions of stability, which represent different strategic behaviors of the MNOs, and study their impact on the equilibriums. For example, the members of a coalition may or may not react when an MNO leaves their group, which in turn affects the decision of the deviator. Such an analysis allows us to dwell on the ramifications of this complex market setting, and in turn design proper regulation methods.

First, we focus on regulation policies that set upper bounds for the market prices. This direct intervention approach is suitable for highly malfunctioning markets that result in very low user utilities. Our analysis provides also insights on how one can identify whether certain market prices are the result of collusion or competition. Accordingly, we investigate regulation policies based on spectrum management. We show that the collusion tendency of the MNOs is heavily affected by their effective capacity (and hence their spectrum). Therefore, by proper tuning this quantity, with respect to other system parameters such as the users population, a regulatory body can act proactively and deter such phenomena.

Summarizing, the *main contributions* of this work are[3]:

i) We develop what we believe is a realistic aspect of massive user behavior in terms of operator selection, having in mind the limited information about user demand and network capacity supply that is common in such markets. For the model above, we derive evolutionary stable equilibrium points.
ii) Having that user model as underlay, we analyze the price competition of the operators and we prove that they have strong incentives to collude and fix their prices. We provide the analytical expression that reveals how their colluding decisions depend on their spectrum and the market size.
iii) We introduce a *coalition formation game* with externalities in order to study these collusion phenomena, and characterize how the price competition among the different coalitions is affected by the market parameters.
iv) We analyze the conditions that render operators collusion beneficial, in terms of revenue. Specifically, we find that as the amount of spectrum that the operators have at their disposal increases, they have

2. The importance of studying collusion and regulation by employing game theoretic models is better exemplified by the 2014 "Nobel Prize" for economics, (or, *Prize in Economic Sciences*) that was awarded to Jean Tirole for his analytical work on *Market power and regulation*. See online http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2014/tirole-facts.html

3. This paper is an extended version of our preliminary work presented in [14], and also builds upon results presented in [15].

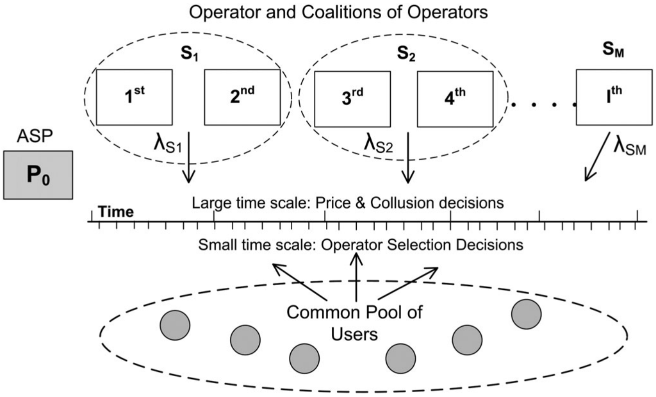


Fig. 2. The market consists of $\mathcal{I}$ operators and $\mathcal{N}$ users. Each user is associated with one operator and each operator can serve concurrently more than one users. Operators may form coalitions, $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_M$ and fix their prices $\lambda_{\mathcal{S}_1}, \lambda_{\mathcal{S}_2}, \ldots, \lambda_{\mathcal{S}_M}$. Users that fail to satisfy their minimum requirements U_0, abstain from the market and select the alternative communication solution P_0.

higher tendency to collude. This leads to higher market distortion where large colluding coalitions are stable.

v) We study the stability of grand coalition or other coalition structures under different *stability criteria* so as to understand the key market parameters that affect such phenomena. For example, we explore how the operators' expectations about each other strategic behavior affects the stability of the collusion structures.

vi) We propose a set of direct (price bounds) and indirect (spectrum allocation) regulation methods so as to effectively deter such phenomena. For example, we identify the minimum amount of spectrum that makes the grand coalition stable. We show that if the allocated spectrum is kept below but closed to this threshold, users get high utility and the effective monopolistic strategies of the operators are deterred.

The rest of the paper is organized as follows. Section 2 introduces the system model, and Section 3 analyzes the competition among the operators. Section 4 analyzes the cooperative game among and explores the stability of the coalitions under different stability criteria. In Section 5 we describe several methods and examples to deter collusion. We discuss related works both from communication networks and economic literature in Section 6, and present our conclusions in Section 7.

2 System Model and Background

We consider a wireless communications services markets (henceforth referred to as *market*) with a large set of users $\mathcal{N} = (1, 2, \ldots, N)$ and a set of MNOs (or, simply *operators*) $\mathcal{I} = (1, 2, \ldots, I)$ as depicted in Fig. 2. Each user may either select one of the $\mathcal{I}$ operators or opt not to purchase services, abstain from the main market, and choose the alternative service provider. The decisions of the users are made based on their perceived satisfaction.

In particular, the net utility of each user who is served by operator $i \in \mathcal{I}$, is:

$$U_i(w_i, n_i, \lambda_i) = \log \frac{w_i}{n_i} - \lambda_i, \tag{1}$$

where n_i is the number of the users served by this specific operator, w_i the total spectrum at its disposal, and λ_i the charged price. Both price and utility are measured in units of currency per unit time. Thus, we will use hereafter the term monetary units (or, shortly MU) to measure these quantities during a certain time interval.[4] For example the price λ_i is the monthly cost of the data plan[5] charged to users of operator i. Note that we model each user's utility as a logarithmic function of the allocated resource in order to represent its perceived satisfaction saturation as the allocated resource increases.[6] Such concave functions are compliant with the economics' principle of diminishing marginal returns, [21], and has been used to model the benefit of users in best-effort wired or wireless networks, [25]. Finally, we assume that the spectrum resource of the operator is *on average* equally shared among its subscribers due to a long-run effect of network management and load balancing. That is, the QoS each user receives, is the same when averaged over time and location.

The system operation is time slotted. In each slot $t = 1, 2, \ldots$, users have the opportunity to change their association and select another operator [23] or decide not to purchase services from any of them. Due to the large number of users and the limited information about the market (i.e., unknown w_i, $\forall i \in \mathcal{I}$), each user updates his choice through an evolutionary process during which he is affected by the choices of other users. In particular, we employ evolutionary game theory (EGT) [16] in order to capture how users are making network selection decisions in this context[7] [17], [19]. However, unlike previous works that employed EGT models to capture user selection, here we employ the respective model proposed in [15] so as to explicitly take into account the out-of-the-market option (for selecting the ASP).

In particular, the probability that a user currently associated with operator i will move to operator j is considered proportional to the utility improvement this choice will bring. Also, it is analogous to the users already associated with operator j, as they essentially act as advertisers of this strategy. In other words, the probability that a user will be informed for a service of an operator depends on the propability to encounter (meet) with another user who is already a subscriber of that operator. The latter probability is of course proportional to the size of the population of this operators subscribers. This model of preferences spreading over random encounters has been extensively used in the literature. (More information on considered evolutionary model is given in Appendix A.1, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2015.2497684. For

4. Note however that our analysis is qualitative and the conclusions do not depend on the time period that these quantities refer to.

5. Although usage-based pricing has been proposed and to some extent adopted in such markets, the current prevalent strategy is to have a certain amount of available mobile data per month, which cannot be transferred to the next time period. This is the modeling approach we follow here.

6. Although a logarithmic utility function is adopted in this work, similar analysis can be done for other concave functions as well.

7. For a detailed discussion supporting the employment of evolutionary game theory in order to model network selection in large user populations, please see [18].

further details please see [33] and [15]). Therefore, we employ the following formula for this probability:

$$p_{ij}(t) = x_j(t)[U_j(t) - U_i(t)]_+, \tag{2}$$

where

$$x_j(t) = n_j/N, \tag{3}$$

is the portion of users already associated with the jth operator at time slot t, and $[\cdot]^+$ denotes the projection on the non-negative orthant. Notice that, for simplicity, we express user utilities as functions with a single argument, i.e., time t.

At the same time, we assume that each user is individually rational and has a reservation utility of U_0 units that must be satisfied in order to agree to pay for the service. This quantity represents the utility the user can ensure by selecting a non-conventional network access option, e.g., a hybrid MVNO [5], [6], that is modeled here by the ASP[8] denoted P_0, Fig. 2. The probability a user switches from operator i to P_0, is:

$$p_{i0}(t) = \gamma[U_0 - U_i(t)]_+, \gamma > 0, \tag{4}$$

where γ is a constant multiplier.

In Appendix A.1, available online, we proved that in this system the evolution of user population that is associated with each operator reaches a stable point which depends on the vector of the prices set by the operators

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_I).$$

Namely, according to the evolutionary game theory framework [33], each user's $i \in \mathcal{N}$ strategy revision protocol which is described by the switching probabilities $p_{ij}(t)$ and $p_{i0}(t)$, yields the following population dynamics:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= x_i(t)[U_i(t) - U_{avg}(t) - x_0(t)(U_i(t) - U_0) \\ &\quad - \gamma(U_0 - U_i(t))_+ + x_0(t)(U_i(t) - U_0)_+], \forall i \in \mathcal{I},\end{aligned} \tag{5}$$

where U_{avg} is the average utility of the market in each slot t, i.e.,

$$U_{avg}(t) = \sum_{i \in \mathcal{I}} x_i(t) U_i(t)\,.$$

The user population associated with P_0 is:

$$\frac{dx_0(t)}{dt} = x_0 \sum_{i \in \mathcal{I}^+} x_i(U_0 - U_i) + \gamma \sum_{j \in \mathcal{I}^-} x_j(U_0 - U_j), \tag{6}$$

where $\mathcal{I}^+$ is the subset of operators offering utility $U_i(t) > U_0$, and $\mathcal{I}^-$ is the subset of operators offering utility $U_i(t) < U_0$, at slot t.

Without loss of generality, in the rest of the paper we consider[9] $U_0 = 0$. Solving the above equations we obtain the stationary state of the system $\boldsymbol{x}^* = (x_1^*, x_2^*, \ldots, x_I^*)$, where x_i^* is the fraction of the user population allocated to operator $i = 1, 2, \ldots, I$. As we proved in Appendix A.2, available online, according to the operators' prices and their amount of spectrum, the stationary conditions satisfy one of the following three cases:

- Case A: $x_i^*, x_0^* > 0$ and $U_i^* = U_0$, $i \in \mathcal{I}$.
- Case B: x_i^*, $x_j^* > 0$, $x_0^* = 0$ and $U_i^* = U_j^*$, with $U_i^*, U_j^* > U_0, \forall i, j \in \mathcal{I}$.
- Case C: x_i^*, $x_j^* > 0$, $x_0^* = 0$ and $U_i^* = U_j^* = U_0$, $\forall i, j \in \mathcal{I}$.

Case A corresponds to the scenario where all operators offer to their clients net utility which is equal to their reservation utility, and some of the users abstain from the market. On the other hand, in case B the market operators offer utility higher than U_0 and hence all users are served by the market. Finally, in case C, all operators offer marginal services equal to U_0, but they have attracted all the users.

Applying the corresponding equations/inequalities for each case, in Appendix A.2, available online, it is shown that the attainable market stationary points x_i^*, $i \in \mathcal{I}$ and x_0^* depend on the prices charged by the market operators and on vector $\boldsymbol{w} = (w_i : i \in \mathcal{I})$ as follows:

$$x_i^* = \begin{cases} w_i e^{-\lambda_i}/N & \text{if } \boldsymbol{\lambda} \in \Lambda_A \cup \Lambda_C, \\ \frac{w_i}{e^{\lambda_i} \sum_{j=1}^{I} w_j e^{-\lambda_j}} & \text{if } \boldsymbol{\lambda} \in \Lambda_B, \end{cases} \tag{7}$$

and

$$x_0^* = \begin{cases} 1 - \sum_{i=1}^{I} w_i e^{-\lambda_i}/N & \text{if } \boldsymbol{\lambda} \in \Lambda_A, \\ 0 & \text{if } \boldsymbol{\lambda} \in \Lambda_B \cup \Lambda_C, \end{cases} \tag{8}$$

where Λ_A, Λ_B and Λ_C are the sets of prices for which a stationary point satisfy case A, B and C respectively:

$$\lambda \in \begin{cases} \Lambda_A & \text{if } \sum_{i=1}^{I} w_i e^{-\lambda_i} < N, \\ \Lambda_B & \text{if } \sum_{i=1}^{I} w_i e^{-\lambda_i} > N, \\ \Lambda_C & \text{if } \sum_{i=1}^{I} w_i e^{-\lambda_i} = N. \end{cases} \tag{9}$$

Clearly, the utilities of all users in any stationary state are equal to each other, and depends on the price region.[10] More specifically, using (1) and (7), it is found that:

$$U_i = \begin{cases} U_0 & \text{if } \boldsymbol{\lambda} \in \Lambda_A \cup \Lambda_C, \\ \log\left(\sum_{j=1}^{I} w_j e^{-\lambda_j}/N\right) & \text{if } \boldsymbol{\lambda} \in \Lambda_B. \end{cases} \tag{10}$$

The dependency of $\boldsymbol{x}^*$ on vector $\boldsymbol{\lambda}$ gives rise to a non-cooperative price competition game, which is described in the next section.

3 Price Competition Among Operators

Each operator anticipates the users strategy and chooses accordingly for each time epoch $\mathcal{T}$ the price that maximizes its revenue. This gives rise to a noncooperative price

8. As we focus on the typical MNOs, we abstract out all these out-of-the-market options with one ASP. Notice also that, from the technical point of view, incorporating P_0 in our model, allows us to calculate exactly how many users are not served by the I operators.

9. All the analysis can be easily adapted for $U_0 > 0$ via a normalization of w_i values (e.g., using w_i/e^{U_0} instead of w_i as described in [15]).

10. Note that, price is selected to be the main parameter to characterize the regions. Because, when thinking of a non-cooperative game in a market with a given $\boldsymbol{w}$ and N, the only parameter that is subject to change is operators prices.

competition game among the operators $\mathcal{G}_{\mathcal{P}} = (\mathcal{I}, \{\lambda_i\}, \{R_i\})$ that is played in the beginning of each epoch:

- Players: the set of the I operators $\mathcal{I} = (1, 2, \ldots, I)$.
- Strategies: every operator $i \in \mathcal{I}$ selects its price $\lambda_i \in [0, \lambda_{max}]$, $\lambda_{max} \in \mathcal{R}^+$.
- Payoff: the revenue function $R_i : (\lambda_i, \lambda_{-i}) \rightarrow \mathcal{R}$, for each operator i.

Each operator $i \in \mathcal{I}$ selects the price that maximizes its revenue

$$R_i = n_i^* \lambda_i = x_i^* N \lambda_i \, .$$

Using (7), the revenue of each operator $i \in \mathcal{I}$ can be expressed as a function of its price λ_i and the prices set by the other operators:

$$R_i(\lambda_i, \boldsymbol{\lambda}_{-i}) = \begin{cases} \dfrac{w_i \lambda_i N e^{-\lambda_i}}{\sum_{j=1}^{I} (w_j e^{-\lambda_j})} & \text{if } \lambda_i < l_0, \\ w_i \lambda_i e^{-\lambda_i} & \text{if } \lambda_i \geq l_0, \end{cases} \tag{11}$$

where we have used:

$$l_0 = \log\left(w_i \Big/ \left(N - \sum_{j \neq i} w_j e^{-\lambda_j} \right) \right), \boldsymbol{\lambda}_{-i} = (\lambda_j : j \in \mathcal{I} \setminus \{i\}).$$

The particular characteristic of this game is that each operator has two different payoff functions depending on the price profile. Despite this characteristic, the payoff function is continuous and quasi-concave as we prove in the Appendix B.1, available online. In the sequel, we analyze the best response of each operator which constitutes a reaction curve to the prices set by the other operators. The equilibrium of the game $\mathcal{G}_{\mathcal{P}}$ is the intersection of the reaction curves of the operators.

Theorem 3.1. *The best response price of an operator i is:*

$$\lambda_i^* = \begin{cases} 1, & \text{if } (1, \lambda_{-i}) \in \Lambda_A, \\ \mu_i^*, & \text{if } (\mu_i^*, \lambda_{-i}) \in \Lambda_B, \\ \lambda_i^C = l_0, & \text{otherwise}, \end{cases} \tag{12}$$

where μ_i^* is the unconstrained optimal point of the upper function case in (49).

Proof of this theorem is given in Appendix B.2, available online.

The important question here is whether there is a solution that concurrently satisfies the best response equations of all operators and if there exists, how can it be found. In other words, we investigate the existence of a Nash equilibrium (NE) for the game $\mathcal{G}_{\mathcal{P}}$. A price vector

$$\boldsymbol{\lambda}^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_{\mathcal{I}}^*),$$

is a NE of $\mathcal{G}_{\mathcal{P}}$, parameterized by the vector $\mathbf{w}$, if it satisfies:

$$R_i(\lambda_i^*, \lambda_{-i}^*, \mathbf{w}) \geq R_i(\lambda_i, \lambda_{-i}^*, \mathbf{w}), \forall i \in \mathcal{I}, \ \forall \lambda_i \geq 0, \forall \boldsymbol{x}^*.$$

Fortunately, the price competition game $\mathcal{G}_{\mathcal{P}}$ is a finite *ordinal potential game* and therefore it has pure Nash equilibria. Therefore, if $\mathcal{G}_{\mathcal{P}}$ is played repeatedly the convergence to the equilibria is ensured through any finite improvement path (FIP) [39]. For detailed proof, please refer to Appendix B.3, available online.

Another important question is whether $\boldsymbol{\lambda}^*$ would be in Λ_A, Λ_B or Λ_C. The answer depends on the spectrum vector, $\mathbf{w}$, and the number of users, N. When the amount of spectrum is scarce, operators would serve less number of users giving them marginal service, and some of the users will stay out of the market, so the stationary condition satisfy case A and $\boldsymbol{\lambda}^* \in \Lambda_A$. As the amount of spectrum increases, operators start serving all of the users but again they only give marginal services and all the users get their reservation utility, so the stationary condition satisfy case C and $\boldsymbol{\lambda}^* \in \Lambda_C$. For high amount of spectrum, operators can offer better services to the users and compete to attract more users. In this case, all the users are served and they get more utility than their reservation utility, so the stationary condition satisfy case C and $\boldsymbol{\lambda}^* \in \Lambda_B$. In the following theorem, we provide NE for a symmetric market, where all the operators have same amount of spectrum, $w_i = w, \forall i \in \mathcal{I}$.

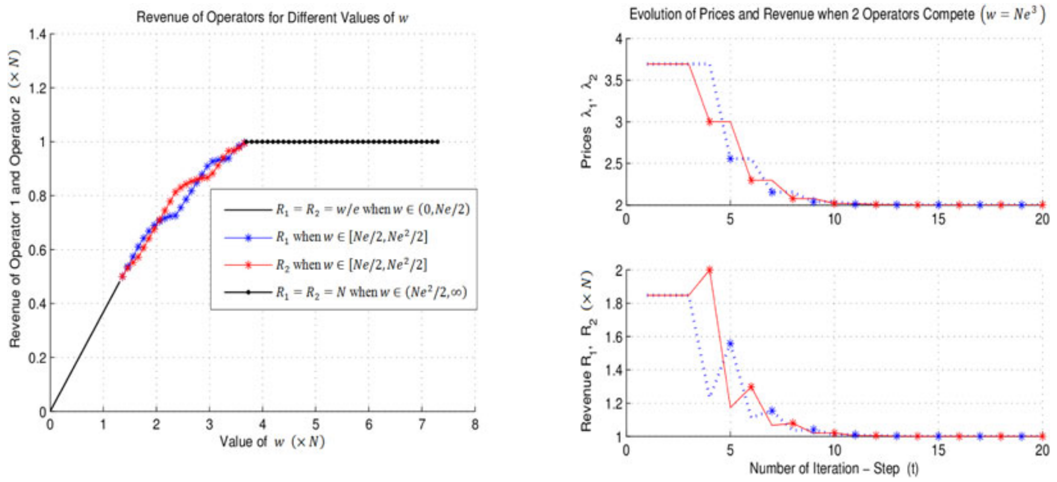
Theorem 3.2. *If the market is symmetric, the non-cooperative game $\mathcal{G}_{\mathcal{P}}$ converges to one of the following pure Nash equilibria:*

- *If $w \in (0, Ne/I)$, there is a unique Nash Equilibrium $\boldsymbol{\lambda}^* \in \Lambda_A$, with $\boldsymbol{\lambda}^* = (\lambda_i^* = 1 : i = 1, 2, \ldots, I)$ and the respective unique market stationary point $\boldsymbol{x}^*$ that satisfies case A.*
- *If $w \in (Ne^{\frac{I}{I-1}}/I, \infty)$, there is a unique Nash Equilibrium $\boldsymbol{\lambda}^* \in \Lambda_B$, with $\boldsymbol{\lambda}^* = (\lambda_i^* = \frac{I}{I-1} : i = 1, 2, \ldots, I)$, which induces a unique respective market stationary point $\boldsymbol{x}^*$ that satisfies case B.*
- *If $w \in [Ne/I, Ne^{\frac{I}{I-1}}/I]$, there exist infinitely many equilibria, $\boldsymbol{\lambda}^* \in \Lambda_C$, which depend on the initial price vector $\lambda(0)$ and each one of them yields a respective market stationary point $\boldsymbol{x}^*$ that satisfies case C.*

Please refer to Appendix B.4, available online for the detailed proof.

Fig. 3a illustrates the revenue of each operator in a symmetric duopoly market, for different values of $w_i = w$. When $w \in (0, Ne/2)$, $\boldsymbol{\lambda}^* = (1, 1) \in \Lambda_A$ and the revenue increases linearly. We observe that when $w \in (Ne/2, Ne^2/2$, the competition of the operators may attain different equilibria $\boldsymbol{\lambda}^* \in \Lambda_C$, depending on the initial prices and on the sequence the operators update their prices. In Fig. 3a, it is assumed that initial prices of both operators are 1.1 MU. When $w \in (Ne^2/2, \infty)$, there is a unique NE where $\boldsymbol{\lambda}^* = (2, 2) \in \Lambda_B$. In this case the revenue of each operator is fixed at $R_i = N$ and does not increase with the amount of spectrum. Fig. 3b illustrates how the NE prices and corresponding revenues converge to NE for the case $w = Ne^3 \in (Ne^2/2, \infty)$. Since the game is proven to be a potential game, for any $w > Ne^2/2$, repeatedly playing the best response strategies will always converge to the same NE independent from the initial prices.

It is obvious that competition yields decreased revenue for the competing operators. Additionally, NE analysis show that in case of high competition, the revenue of operators does not increase even if they make further CAPEX investments and increase—for example—their spectrum w_i. *This fact suggests that operators may decide to collude and jointly*



(a) The outcome of the operators competition ($\mathcal{G_P}$ equilibrium) for different values of parameter w, i.e. in different intervals.

(b) Evolution of operator competition for $w = Ne^3 \in (Ne^2/2, \infty)$. The game is played repeatedly and operators updated myopically their price based on the previous strategy of the other operators.

Fig. 3. Operators revenue and competition for different values of w.

determine their pricing strategy in the market, in order to increase their revenue. In the next section, we employ coalitional game theory and study collusion of operators in the context of the market of Fig. 2.

4 Collusion of Operators

We focus on the class of collusion strategies according to which all colluding parties agree to set a common price. Namely, assume that a group of operators form a coalition, denoted with $\mathcal{S}_k$, $\mathcal{S}_k \subseteq \mathcal{I}$, and set the same price $\lambda_{\mathcal{S}_k}$, as it is shown in Fig. 2. Notice that the coalition members still offer different quality of service since they may have different spectrum at their disposal. However, after a certain period of time, the evolving decisions of the users will eventually yield a user distribution where each member operator offers the same net utility to its subscribers. Therefore, within each coalition, the users are allocated to each operator in proportion to its spectrum.

When there are more than one coalitions, they compete with each other on selecting the prices that will yield the higher revenues for them. Clearly, the mechanics of this competition are the same as when different independent operators compete with each other to attract users.[11] In this context, two basic parameters that describe every coalition $\mathcal{S}_k$ are *(i)* the number $|\mathcal{S}_k|$ of participating operators and, *(ii)* the sum of the spectrum at their disposal, $W_{s_k} = \sum_{i\in\mathcal{S}_k} w_i$. In other words, a coalition acts as a single operator with W_{s_k} amount of spectrum. Hence, according to (49) the revenue of operator i that participates in coalition $\mathcal{S}_k$, is:

$$R_i(\lambda_{\mathcal{S}_k}) = \begin{cases} \dfrac{w_i\lambda_{\mathcal{S}_k}N}{W_{s_k} + e^{\lambda_{\mathcal{S}_k}}\sum_{j\in\mathcal{I}\setminus\mathcal{S}_k} w_j e^{-\lambda_j}} & \text{if } \lambda_{\mathcal{S}_k} < l_0(\mathcal{S}_k), \\ w_i\lambda_{\mathcal{S}_k} e^{-\lambda_{\mathcal{S}_k}} & \text{if } \lambda_{\mathcal{S}_k} \geq l_0(\mathcal{S}_k), \end{cases} \tag{13}$$

11. A critical observation in order to see this is that the revenue of each operator within each coalition increases as the revenue for the entire coalition increases.

where we have used

$$l_0(\mathcal{S}_k) = \log\left(W_{s_k} \Big/ \left(N - \sum_{j\in\mathcal{I}\setminus\mathcal{S}_k} w_j e^{-\lambda_j}\right)\right),$$

that depends on the specific coalition $\mathcal{S}_k$. Apparently, this revenue depends on the prices set by the operators that do not belong in $\mathcal{S}_k$, $\boldsymbol{\lambda}_{-\mathcal{S}_k}$.

The non-cooperative game among the $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_K\}$, $K > 1$ coalitions is identical to the price competition game $\mathcal{G_P}$ with K operators where $w_k = W_{s_k}$ for $k = 1, 2, \ldots, K$. Therefore, the optimal price $\lambda^*_{\mathcal{S}_k}$ for each coalition $\mathcal{S}_k$ can be found by using equation (12), after substituting i with $\mathcal{S}_k$, w_i with W_{s_k} and $\boldsymbol{\lambda}_{-i}$ with $\boldsymbol{\lambda}_{-\mathcal{S}_k}$. A price vector:

$$\boldsymbol{\lambda}^* = (\lambda^*_{\mathcal{S}_1}, \lambda^*_{\mathcal{S}_2}, \ldots, \lambda^*_{\mathcal{S}_K}),$$

is a Nash Equilibrium when operators in all coalitions set their best response prices ($\lambda^*_{\mathcal{S}_k}$), taking into account the decisions of the other operators. Since this game is identical to $\mathcal{G_P}$ which is proven to be a potential game, existence of pure NE is guaranteed and the game would converge to NE when operators adopt best response prices sequentially. The revenue of each operator $i \in \mathcal{S}_k$ in the NE, R_i^*, can be computed by substituting NE prices to the function R_i given in (70). Here we stress that, according to the previous analysis, the price and revenue of each operator at the NE, depends not only on the size of its coalition, but also on the structure of the coalitions that are formed by the other operators. This *externality* affects the strategy of operators.

On the other hand, when all operators collude and form one single coalition, the so-called *grand coalition*, i.e., $S_1 = \mathcal{I}$, they set the same price $\lambda_{\mathcal{I}}$, and each one of them has revenue:

$$R_i(\lambda_{\mathcal{I}}) = \begin{cases} \dfrac{w_i\lambda_{\mathcal{I}}N}{\sum_{j=1}^{I} w_j} & \text{if } \lambda_{\mathcal{I}} < \log\left(\sum_{j=1}^{I} w_j/N\right), \\ w_i\lambda_{\mathcal{I}} e^{-\lambda_{\mathcal{I}}} & \text{otherwise.} \end{cases} \tag{14}$$

Notice that operators within the same coalition receive different revenue due to their different amounts of spectrum. When the *grand coalition* is formed, operators do not compete with each other, but rather they cooperate and act as one single operator. This results in the emergence of a monopolistic market where the optimal price, $\lambda_{\mathcal{I}}$, depends only on w_i:

$$\lambda^*_{\mathcal{I}} = \begin{cases} \log\left(\sum_{i=1}^{I} w_i/N\right) & \text{if } \log\left(\sum_{i=1}^{I} w_i/N\right) > 1, \\ 1 & \text{otherwise.} \end{cases} \quad (15)$$

Note that utility of users are equal to U_0 in case of grand coalition. This observation summarizes how the collusion impacts the net utility perceived by the users. That is, the grand coalition can increase its common price and extract all the utility of the users up to their reservation utility. Such a market has only a minimum welfare of $N \cdot U_0$ units.

Now we ask the following questions: *(i)* do operators have an incentive to collude and create an oligopolistic or even a monopolistic market? *(ii)* Under which conditions does collusion increase the revenue of operators? The answers to these questions are significant from a regulatory perspective, since they enable the derivation of methods that promote competition in this kind of markets.

4.1 Coalitional Game for Colluding Operators

Coalition formation games in economic setups with externalities are modeled in *Partition Function Form* which was introduced in [34]. Given a partition Π of $\mathcal{I}$ and a coalition $S_k \in \Pi$, the pair $(S_k; \Pi)$ is called an embedded coalition of $\mathcal{I}$. The set of all embedded coalitions is denoted by $EC(\mathcal{I})$. We model the operators coalitional game, $\mathcal{G}_{\mathcal{C}} = (\mathcal{I}, v)$, in partition function form, as follows:

- The players are the operators $\mathcal{I} = (1, 2, \ldots, I)$.
- $\phi_i(\mathcal{S}_k; \Pi) = R^*_i$ is the equilibrium payoff of any operator i in coalition $\mathcal{S}_k$ within a partition Π.
- $v(\cdot)$ is the function that assigns to each embedded coalition $(\mathcal{S}_k; \Pi) \in EC(\mathcal{I})$, a set of values $\{\phi_1(\mathcal{S}_k; \Pi), \ldots, \phi_{|\mathcal{S}_k|}(\mathcal{S}_k; \Pi)\}$.

We assume that there is no central authority coordinating the interactions of operators, and that each operator is a selfish revenue-maximizing entity. Therefore, coalition formation is accomplished in a distributed fashion. In a distributed coalition formation algorithm, there are three main components [36], [32]: (1) well-defined orders suitable to compare collections of coalitions, (2) two rules for merging and splitting coalitions, and (3) adequate notions for assessing the stability of a partition. In the sequel, we describe these components for the operators coalition game $\mathcal{G}_{\mathcal{C}}$.

Coalitional games are classified as transferable utility (TU) and non-transferable utility (NTU) games. In TU games, the actions taken by the coalitions determine the payoff of each coalition. The members of the coalition then need to divide this joint payoff. On the other hand, in NTU games, the actions taken by the coalitions determine each player's payoff. $\mathcal{G}_{\mathcal{C}}$ is defined as an NTU game, because utility transfer agreements and revenue sharing are mostly prohibited by laws. Therefore, rather than the aggregate utility of the players in a coalition, we have to consider individual utilities. We use Pareto order [36] as a comparison metric between two collections of coalitions. Given two collections $\mathcal{S}$ and $\mathcal{T}$, $\mathcal{S}$ is preferred over $\mathcal{T}$ by Pareto order if at least one player in $\mathcal{S}$ improves his payoff without hurting other players. We define merging and splitting rules for forming and breaking coalitions, according to Pareto order, as follows:

Merge rule. Any collection of disjoint coalitions $\mathcal{S} = \{\mathcal{S}_1, \ldots, \mathcal{S}_K\}$ can agree to merge into a single coalition $\mathcal{T} = \cup_{k=1}^{K} \mathcal{S}_k$, if the new coalition $\mathcal{T}$ is preferred by all the operators over the previous collection of coalitions $\mathcal{S}$ according to the Pareto order. In other words, $\mathcal{T}$ is preferred over $\mathcal{S}$ if at least one operator in $\mathcal{S}$ improves its revenue without decreasing the revenue of any other operator.

Split rule. A single coalition $\mathcal{T}$ splits into a collection of disjoint coalitions $\mathcal{S} = \{\mathcal{T} - \mathcal{R}, \mathcal{R}\}$ if operators in coalition $\mathcal{R}$ prefer $\mathcal{S}$ over $\mathcal{T}$ according to the Pareto order. Here, we assume that one or more operators can leave the coalition and form a new coalition if it is to their advantage, although this may have an impact on the revenue of other operators in the initial coalition.

Based on these rules, the game $\mathcal{G}_{\mathcal{C}}$ has certain properties that are summarized in the next theorem.

Theorem 4.1. *The game $\mathcal{G}_{\mathcal{C}}$ is a coalition formation game with non-transferable utilities, having the following properties:*

1) *If the NE price vector $\boldsymbol{\lambda}^* \notin \Lambda_B$, there is no incentive for operators to collude.*[12]
2) *It exhibits* positive *externalities, i.e., a merger between two coalitions always make other coalitions better off.*
3) *Operators in coalition $(S_i; \Pi)$ get lower revenue per unit of spectrum, compared to the operators in coalition $(S_k; \Pi)$, if $W_{s_i} > W_{s_k}$.*
4) *It is not always super-additive, with respect to merging and the resulting revenue of the operators.*
5) *When two or more coalitions merge, the equilibrium prices of all operators increase. There is also a direct proportionality between aggregate revenues of operators in a coalition and their prices.*

Proof. Please see Appendix C, available online. □

The above theorem provides a number of very important insights regarding the collusion behavior of the mobile network operators. First, Property 3 along with Property 2 indicate that some operators may *free ride*: do not participate in a coalition, act independently, and gain higher revenue due to the collusion of other operators. Moreover, Property 5 suggests that operators have higher incentive to collude when there is no price upper bound.

Furthermore, Property 4 suggests that there are some cases where operator(s) in a coalition cannot gain more by merging with another coalition. This is due to the fact that operators with small amount of spectrum may obtain lower revenue when merging with a coalition with large W_{s_k}. Although the aggregate revenue of the coalition increases

12. Note that this property is very intuitive because when $\boldsymbol{\lambda}^* \in \Lambda_A \cup \Lambda_C$, users already get their minimum utility of U_0. It is not possible to do any price-fixing that can further decrease the utilities of the users, and increase the revenues of the operators. Therefore, in the rest of this study, we restrict our analysis to the cases where $\boldsymbol{\lambda}^* \in \Lambda_B$.

(after merging), the individual share of the newly merging operator may decrease (due to Property 3).

4.2 Stability Analysis

Since we allow the operators to dynamically join or leave a coalition, it is very important to understand whether the above equilibriums are stable or not. Moreover, these stability results depend largely on the employed stability notion, which, in turn, captures how rational, elaborate or risk averse, the market players are. Namely, in games with externalities, when a group of players decides to deviate, they need to anticipate and take into account the reaction of the external (i.e., non-members) players. Different assumptions of the behavior of external players give rise to different notions of stability. In the coalitional game theory literature (such as [37] and [38]), there exist various notions of stability depending on the underlying hypothesis regarding the reaction (or, strategy) of the external players. This reaction is mainly related to each player's strategy.

The most restricted stability concept is *core stability*. A partition Π is said to be *core-stable*, if no group of players has an incentive to deviate, even if they consider that external players react in such a way as to maximize the payoff of deviators. In this case, players are assumed to be too optimistic and they have a high tendency to deviate, so it is difficult to have a stable coalition structure. Another widely used stability concept is *δ-stability*, where external players are expected to not change their status. Obviously δ-stability is weaker than core-stability, i.e., a δ-stable coalition may not be core-stable, since operators have more tendency to stay in their coalitions.

Another more natural expectation of the deviating players is that external players will take their deviation for granted and try to maximize their own payoff through merge and split operations. This is called rational expectations [35] and the respective stability notion can be referred to as *r-stability*. In markets where operators have less interaction with each other, it might be expected that external operators do not change their status, and γ-stability could be more reasonable stability concept. On the other hand, in markets where operators have high interaction, r-stability could be more appropriate.

In this paper, we define deviation of a group of players as simply merging or splitting. We study the stability of coalition structures, and obtain the following results:

Lemma 4.2. *Grand coalition is core-stable, δ-stable and r-stable if the following condition holds:*

$$\phi_k(\mathcal{I}) \geq \phi_k(\{k\}; \{\{k\}, \mathcal{I} - \{k\}\}),\ \forall k \in \mathcal{I}. \quad (16)$$

In other words, grand coalition is core-stable, δ-stable and r-stable if none of the operators can gain more by defecting while the other operators do not dissolve. If the condition (16) does not hold, the grand coalition is neither core-stable nor δ-stable. However, it can be still r-stable, since after defection of an operator, another rational operator may also decide not to stay in the coalition. This may reduce the revenue of the first defecting operator, probably to lower level than its initial revenue. Hence, an operator with rational expectations will not defect from the grand coalition in such cases. One example is given in the next section (see Fig. 4).

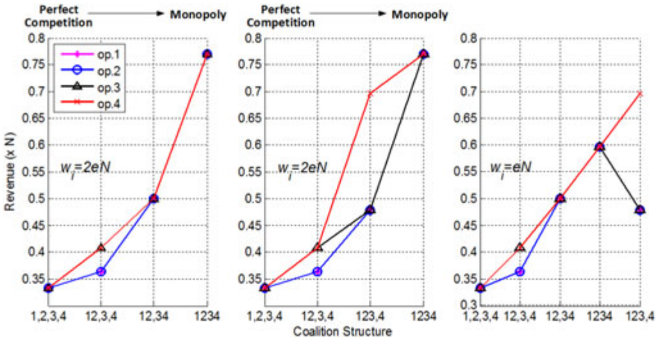


Fig. 4. Operators revenue during the coalition formation process. In the first two figures (from left to right), it is $w_i = 2eN$, $i \in 1, 2, 3, 4$ for all operators, while in the last figure, it is $w_i = eN$. The *x*-axis shows the formed coalitions (e.g., 12 stands for coalition of operator 1 and operator 2), and the *y*-axis presents the operators' revenue per user. Initially all the operators act independently. The first two figures show the evolution from full competition to monopoly. In the third figure, the grand coalition is not stable.

Lemma 4.3. *A partition Π is core-stable if and only if the following conditions hold:*

- *None of the operators or coalitions has a tendency to merge,*
- *For any coalition $S \in \Pi$:*

$$\phi_k(S; \Pi) \geq \phi_k(\{k\}; \{\{k\}, \mathcal{I} - \{k\}\}),\ \forall k \in S. \quad (17)$$

This lemma generalizes core-stability requirement for any coalition structure. Coalition structures other than grand coalition can also be stable, but an obvious requirement is that none of the operators should have a tendency to merge. According to (17), such a coalition structure is core-stable if none of the operators gain more by deviating even if all the other operators merge after its deviation, which is the best possible scenario for it. Proofs of Lemma 4.2 and Lemma 4.3 are given in Appendix C, available online.

On the other hand, a coalition structure with less optimistic operators is δ-stable if none of the operators has a tendency to defect in case other operators do not change their status. Similarly, a coalition structure is r-stable if none of the operators has a tendency to defect in case other operators change their status only in a rational manner so as to increase their revenues. A core-stable coalition structure is always δ-stable and r-stable, while a δ-stable coalition structure may not be r-stable, and an r-stable coalition structure may not be δ-stable. This is because after defection of an operator, there are some cases where other rational operators/coalitions may want to merge, or they may want to split. In former cases, operators with rational expectations would have an incentive to deviate, since the merger of external operators will increase its utility due to Property 3 (so γ-stable structure may not be r-stable). Similarly, latter cases would discourage deviation of operators with rational expectations, so although a structure may not be γ-stable, it can be r-stable.

Finally, there are some cases where none of the coalition structures are core-stable, δ-stable or r-stable. In the next

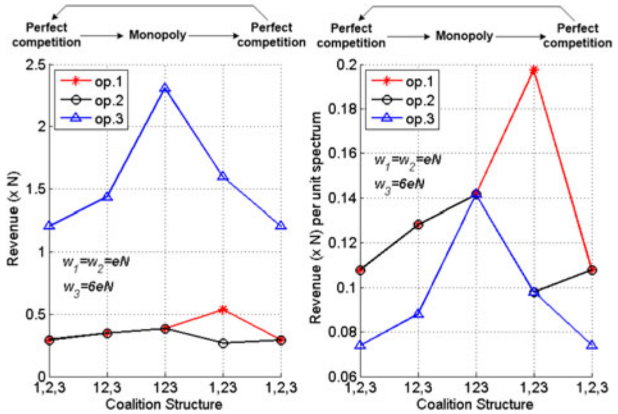


Fig. 5. The first figure illustrates revenues of three operators for evolving coalition structures. The second figure illustrates the revenue per unit spectrum for the same case. The first two operators have the same amount of spectrum ($w_1 = w_2 = eN$), while the third one has six times more spectrum. In this scenario, none of the coalition structures are core-stable or δ-stable, while grand coalition is r-stable.

section, we will give some examples to illustrate these cases. Existence and structural characteristics of stable coalitions are of interest from the regulatory perspective. More importantly, using the different stability notions, a central planner can identify how likely is to have collusion phenomena (under different assumptions about MNOs' strategic behavior), and how stable these would be. Such conclusions lay the foundations for undertaking corrective actions for alleviating such market distortion phenomena. In Section 5, we study several regulation methods for preventing collusion.

4.3 Numerical Examples

For the sake of clarity, first we consider a symmetric market with four operators, where each operator has the same amount of spectrum. In Fig. 4, the first two plots show two coalition formation stages when $w_i = 2eN$. In both cases, grand coalition is the only stable coalition (in any stability sense out of the ones above), since it provides highest possible revenue for all operators. This result coincides with the fact that condition (16) is satisfied.

The third plot in Fig. 4 shows coalition formation when $w_i = eN$. The output of the coalition game remains the same as in the previous case for all coalition structures except the grand coalition. Condition (16) is not satisfied and grand coalition is not core-stable, δ-stable and r-stable. One of the operators can increase its revenue by leaving the coalition if the other operators act rationally and do not split after its defection.

Next, we consider an asymmetric market with three operators having spectrum $w_1 = w_2 = eN$ and $w_3 = 6eN$. Fig. 5 shows the coalition formation steps. In this example, the grand coalition is not core-stable or δ-stable, since operator 1 can gain more by defecting, given that the other operators not split after its defection. However, if operator 1 has rational expectations, it would expect that, after its defection, operator 2 would not stay in the coalition with operator 3 any more in order to increase its revenue, and act alone. In that case, operator 1 would not gain more than its revenue in the grand coalition. Therefore, it would not defect and the grand coalition is r-stable. Another observation from Fig. 5 is that none of the coalition structures are core-stable or δ-stable.

The second plot in Fig. 5 illustrates the revenues per unit spectrum, instead of total revenues of operators. One observation is that, operators with smaller w_i value gain more revenue per unit spectrum than operators with higher w_i values. It is also clearly visible how operators with smaller w_i values are apt to defect from large coalitions and free ride. It can be easily verified that all the examples given in this section support the properties defined in Theorem 4.1.

5 Market Regulation

Stability of the grand coalition or existence of a few number of large coalitions leads to a monopolistic or highly oligopolistic market, which implies significantly low social welfare. Existence of a regulating agency can reduce the effects of this situation and prevent stability of big coalitions. In this section, we discuss several methods for regulating the market, in the sense of enforcing a specific desired operator coalition structure (or, equivalently, in the sense of discouraging certain undesired coalitions).

Firstly, it is important to note that all the analysis in the previous section can be employed so as to identify collusion. For example, if the prices adopted by the operators are identical and close to λ_I^* value given in (15), then it can be concluded that the identical prices are result of a price-fixing agreement of a grand coalition. Similarly, other coalition structures can be identified by investigating market parameters (such as N and w_i) and adopted prices. Identifying collusion is important to keep down its negative effects. Now, we turn our attention to preventing collusion, and describe several methods.

5.1 Enforcing Price Upper Bound

According to Property 5, when operators collude, their NE prices increase and there is a direct proportionality between aggregate revenue of operators and their prices. Therefore one natural way to reduce the benefit of collusion is to enforce an upper bound on the prices, λ_{max}. If the NE price of an operator (or a coalition) in a market without such a price bound is above λ_{max}, it is exactly equal to λ_{max} in a market with price upper bound. Benefit reduction is more evident for big coalitions, since their NE prices are higher. As an example, if λ_{max} is set to 2.5 MU in a symmetric market with four homogeneous operators where $w_i = 2eN$ (as in the first two plots of Fig. 4), NE price of an operator in grand coalition reduces to 2.5, while NE prices in other coalition structures do not change. As a result, revenue of an operator in the grand coalition reduces from $0.77N$ to $0.62N$, and renders the latter unstable.

Now, let us define the *size* of a coalition as the number of operators in that coalition. For the sake of clarity, we consider the symmetric market case, and we define a new set $\mathcal{L}$, which includes sizes of all coalitions. For example, for a grand coalition of four operators, $\mathcal{L} = \{4\}$, for the full competition case, $\mathcal{L} = \{1, 1, 1, 1\}$, for a coalition structure with two coalitions, $\mathcal{S}_1$ and $\mathcal{S}_2$, where $|\mathcal{S}_1| = |\mathcal{S}_2| = 2$, $\mathcal{L} = \{2, 2\}$, and so on. We have to note that this representation is only valid for symmetric market case, where all the operators own equal amount of spectrum, and thus coalition structures with the same coalition sizes are actually identical.

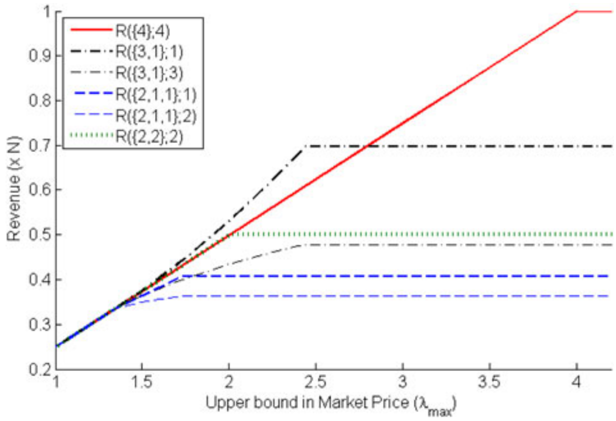


Fig. 6. Revenues of the operators in different coalition structures for different values of λ_{max}. There are four operators, each with $w_i = 5eN$. The revenue of an operator in coalition $\mathcal{S}_i$ is shown by $R(\mathcal{L}; |\mathcal{S}_i|)$, where $\mathcal{L}$ is the set of coalition sizes, and $|\mathcal{S}_i|$ is the number of operators in $\mathcal{S}_i$. For example, $\mathcal{L} = \{3,1\}$ means that there exists a coalition of three operators and an operator acts alone. $R(\{3,1\};1)$ stands for the revenue of the operator acting alone in such a coalition structure. On the other hand, $R(\{3,1\};3)$ stands for the revenue of any of the other three colluding operators. Similarly, $R(\{4\};4)$ stands for the revenue of any operator in the grand coalition.

Fig. 6 illustrates the revenue of any operator in various coalition structures for different values of λ_{max} in a symmetric market with $w_i = 5eN$. It is observed that grand coalition is stable (in any stability notion) when $\lambda_{max} > 2.78$ MU. However, when 2.0 MU $< \lambda_{max} \leq 2.78$ MU, grand coalition is not r-stable or δ-stable anymore since one of the operators will have an incentive to free-ride. Coalition structures with $\mathcal{L} = \{3,1\}$ are δ-stable, because none of the operators can gain more by defecting, while the others remain as are. However, it is not r-stable either, because an operator can gain more by deviating from the coalition, while expecting that the other free-rider is rational and will merge with it to increase their revenues. Coalition structures with $\mathcal{L} = \{2,2\}$ are not δ-stable or r-stable, because operators can gain more if they merge and form the grand coalition. Other coalition structures with more than three coalitions are not saturated to merge operation, so they are also not stable. Hence, we can say that there is no core-stable or r-stable coalition structure if 2.0 MU $< \lambda_{max} \leq 2.78$ MU.

Similar conditions hold for 1.78 MU $< \lambda_{max} \leq 2.0$ MU, except that coalition structures with $\mathcal{L} = \{2,2\}$ are now δ-stable, because operators cannot gain more by merging and forming the grand coalition. However, they are not r-stable, because one of the operators may deviate, expecting that the other three merge in order to increase their revenues. Thus, the deviating operator can gain more as a free-rider.

For 1.33 MU $< \lambda_{max} \leq 1.78$ MU, coalition structure with a single free-rider ($\mathcal{L} = \{3,1\}$) is not δ-stable anymore, since a second operator will also have an incentive to free ride. Moreover, coalition structure with $\mathcal{L} = \{2,2\}$ is not only δ-stable but also r-stable in this case. This is because none of the operators can gain more by deviating, since external operators are not expected to merge if they act rationally. Also they can not gain more by merging. On the other hand, none of the coalition structures are core-stable for this case as well.

For very small values of λ_{max}, i.e. $\lambda_{max} \leq 1.33$ MU, NE prices are equal to λ_{max} for all coalition structures and revenue of the operators are the same, therefore all coalition structures are stable. Stable coalitions structures for different values of λ_{max} are summarized in Table 1.

TABLE 1
Stable Coalition Structures for Four Operators with $w_i = 5$eN for Different Values of λ_{max}

λ_{max} range	core-stability	δ-stability	r-stability
(2.78,inf)	{4}	{4}	{4}
(2,2.78]	not exists	{3,1}	not exists
(1.78,2]	not exists	{3,1}, {2,2}	not exists
(1.33, 1.78]	not exists	{2,2}	{2,2}
(0, 1.33]	all	all	all

Fig. 7 illustrates the utility of users in market equilibrium for different λ_{max} values, under δ-stable coalition structures. When 1.78 MU $< \lambda_{max} \leq 2.0$ MU, there are two different δ-stable coalition structures ({3,1} and {2,2} as shown in Table 1), so the utilities for both coalition structures are shown for this range. By adjusting the λ_{max} value, a regulator can achieve a desired market outcome, in terms of user utilities and/or operator revenues.

5.2 Determining the Amount of Spectrum

The amount of spectrum allocated to operators increases, operators have more incentive to form coalitions. Therefore, it is possible to prevent stability of big coalitions by determining the amount of spectrum w_i each operator has at its disposal. We have already shown an example in Fig. 3, where reducing w_i value from $2eN$ to eN prevent the stability of the grand coalition.

Fig. 8 shows the maximum amount of spectrum per operator for preventing stability of grand coalition, in the symmetric market case. For example, in case of four operators, grand coalition is stable when each operator has more than $4.06N$ amount of spectrum. For $w_i \leq 4.06N$, grand coalition is not stable, so $w_i = 4.06N$ is the threshold value for preventing monopoly. Threshold values for different number of operators are obtained according to Lemma 4.2 and plotted in Fig. 8. As the number of operators increase, threshold value increases faster and it is easier to prevent stability of grand coalition.

Now, we turn our attention to the market outcome for different amount of spectrum allocated to operators. Market

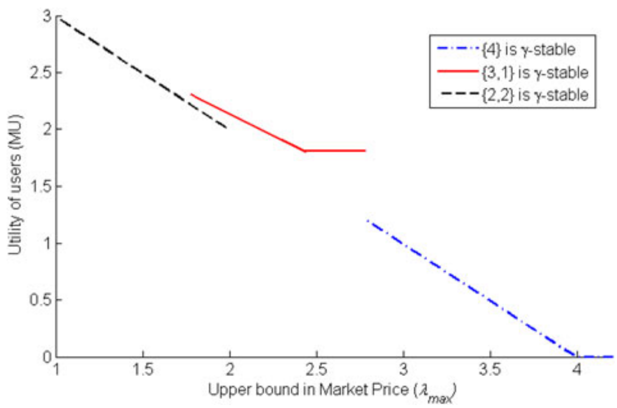


Fig. 7. Utility of users in δ-stable market with four operators ($w_i = 5eN$) for different values of λ_{max}.

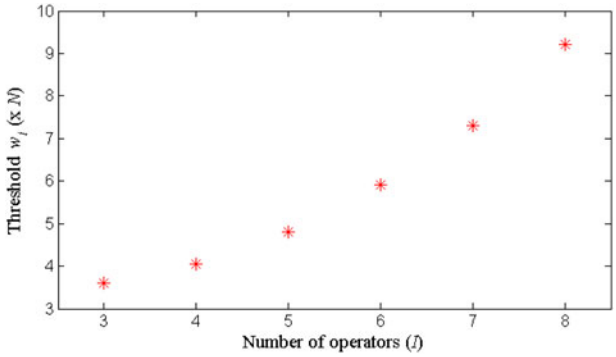


Fig. 8. Maximum amount of spectrum per operator (threshold w_i) to prevent stability of grand-coalition.

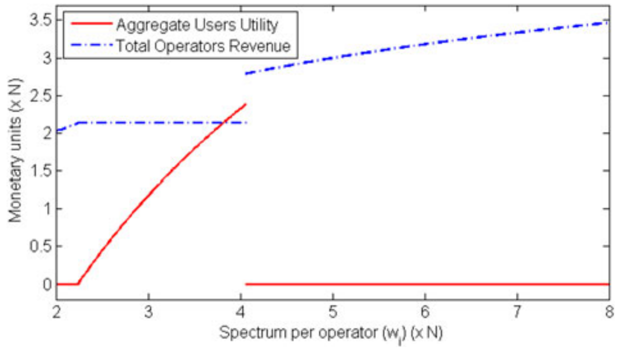


Fig. 9. Aggregate utility of users and total revenue of operators for different amounts of spectrum per operator (w_i), in a symmetric market of four operators. Given values are for the core-stable coalition structures. Grand coalition is stable for $w_i > 4.06N$. For $w_i \leq 4.06N$, coalition structure with $\mathcal{L} = \{3, 1\}$ is the stable coalition structure. Both functions discontinue around the threshold value since both the utility of users and the revenue of operators are different under different coalition structures of operators.

outcome can be measured in different ways. We consider two metrics: (i) Aggregate utility of users, and (ii) Total revenue of operators. Fig. 9 illustrates how these two values change for different amount of spectrum per operator, in a symmetric market with four operators. As we show in Fig. 8, threshold w_i value for four operators is approximately $4.06N$. For w_i values larger than $4.06N$, grand coalition is core-stable. In case of grand coalition, operator revenues are high, but the users get zero utility. For w_i values lower than the threshold, coalition structures with $\mathcal{L} = \{3, 1\}$ are core-stable according to Lemma 4.3. As we further reduce w_i, utility of users decreases but revenue of operators stays same as long as NE price vector, λ^*, is in Λ_B. For very low values of w_i, $\lambda^* \in \Lambda_C$ and users get zero utility in the market equilibrium. It is interesting to note that if w_i is less but close to the threshold value, users get high utilities while monopoly is avoided. Further decreasing w_i is not beneficial for both the users and the operators.

6 Related Work

The importance of regulating communications market has long been considered as a necessary means due to collusion and monopoly phenomena [12], [13]. Most of the related works focus on wireline networks and/or on collusion strategies over interconnection charged prices. Besides, many of the existing approaches are high level and do not consider the specifics of wireless communication networks, such as the heterogeneity of the different co-existing networks. More detailed models have been studied in the context of network economics, [21], [22], [24], and only recently for wireless services markets where operators strive to attract subscribers [25], [26], [27]. However, these latter works do not study regulation and collusion, nor consider the option of the alternative service provider.

An interesting instance of such heterogeneous markets is when mobile virtual network operators co-exist and compete with the traditional mobile network operators, e.g., see [28] and references therein. An interesting question in that case is how the MVNOs compete with the MNOs, given that they need to buy in wholesale prices bulk access from the latter. In many cases though, the competition yields an undesirable outcome for the sellers. For example, in [29] the authors have shown that selfish pricing strategies of ISPs may decrease their accrued revenue. Similarly, in our previous work [15] we have established that under certain conditions the price competition of wireless operators may yield decreased revenue for them. Therefore, it is reasonable to assume that they have incentives to collide instead of competing with each other (even if they are of different type).

The analysis of collusion phenomena in such markets remains unexplored although operators very often collude in terms of price fixing [10], [9], [8]. Previous works focused on general network models, e.g., [30], or analyzed collusion from the perspective of the operators without modeling the detailed strategy and behaviors of the users, [25], [31]. In part, this shortage of works is due to the complexity and intractability of coalitional game theoretic models [32].

In this work, we restrict our analysis to the case where operators collusion is realized by adopting identical prices, partly due to the covert nature of this activity. This way, we are able to use a rigorous model, based on coalitional game theory, while decreasing the complexity of the analysis and maintain its tractability. Moreover, we do not restrict our study to the stability investigation of the grand coalition, as it is the case in the vast majority of related works (e.g., [31]). On the contrary we analyze the non-cooperative interaction (competition) between different coalitions by using the theory of coalition formation games, under different assumptions about the underlying strategic MNO behavior, which leads to different notions of coalitions stability.

Another crucial point is the network selection process followed by the users, based on the advertised prices and the quality of the offered services. For example, [20] considered this problem for wireless networks recently and derived the Nash equilibriums for users association, based on the assumption of complete information. On the contrary, our setting involves large populations and limited information about the actual quality of services (prices are considered globally available). Therefore, the users make myopic network selection decisions, and update them periodically, based on the net utility they expect to receive in a new network. Such information comes often from the users that are already associated with the latter.

Finally, unlike other works that considered such evolutionary game models, e.g., [17], [18] and [19], we explicitly allow a user to opt for an alternative network solution. This modeling option is of crucial importance today due to the abundance of communication solutions, and changes radically the obtained equilibriums.

7 Conclusions

We have presented a wireless communication services market where a set of mobile network operators offer services to a large population of users. Using an evolutionary game model, we captured the users dynamics in terms of operator selection, in the presence of limited market information, and alternative out-of-the-market solutions. We studied the price competition among the different operators and identified the conditions that render highly beneficial for them the price-fixing collusion. Accordingly, we defined a coalition formation game in partition function form, and studied if, when, and which coalition structures are stable under various stability notions. We characterized the impact of the amount of spectrum owned by each operator to the stability of a *de-facto* monopoly, and we introduced a framework for regulation methods that can spur competition. Likewise, we provided mathematical tools to analyze how an upper bound on the prices, that is set based on the properties of the market, helps to prevent collusion of operators.

Collusion and price-fixing is against the law in many countries, yet the history shows that law enforcement measures against them are often quite ineffective. Our results, which quantify the motivation for building such coalitions, may lead to tangible solutions towards identifying colluding operators in the market. Such regulatory schemes and their analysis constitutes a promising research direction.

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